Monopoly Pricing when Consumers are Antagonized by Unexpected Price Increases: A "Cover Version" of the Heidhues-Koszegi-Rabin Model*

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Abstract

This paper reformulates and simplifies a recent model by Heidhues and Kőszegi (2005), which in turn is based on a behavioral model due to Kőszegi and Rabin (2006). The model analyzes optimal pricing when consumers are loss averse in the sense that an unexpected price hike lowers their willingness to pay. The main message of the Heidhues-Kőszegi model, namely that this form of consumer loss aversion leads to rigid price responses to cost fluctuations, carries over. I demonstrate the usefulness of this "cover version" of the Heidhues-Kőszegi-Rabin model by obtaining new results: (1) loss aversion lowers expected prices; (2) the firm's incentive to adopt a rigid pricing strategy is stronger when fluctuations are in demand rather than in costs.

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1 Introduction

Like other creative artists, economic theorists value originality. When we construct a model of a certain economic phenomenon, we try to distance and differentiate our personal creation from previous work, unless we wish to present it as an extension, application or foundation of an existing model. However, there is also value in a different sort of exercise, in which the theorist takes an existing model and tries to rewrite it from scratch. When playing this game, the economic theorist steps into the shoes of another theorist, takes her economic motivation and basic modelling idea as given, but then tries to "do it his way". The result of such a re-modelling exercise can be viewed as a "*cover version*" of the original model. It is not an entirely new creation, because it is a deliberate and explicit variant on an existing model. However, it is also not entirely derivative, because the variant is often sufficiently different to merit separate attention. By offering a different way of formalizing the same economic idea, the cover version contributes to our understanding of this idea. Furthermore, the cover version may be technically easier to apply in certain domains, in which case it expands the original model's scope of applications.

This paper proposes a "cover version" of a recent model of optimal pricing when consumers are loss averse, due to Heidhues and Kőszegi (2005) - which in turn builds on a behavioral model proposed by Kőszegi and Rabin (2006). These two papers are referred to as HK and KR in the sequel, and the model in its totality is referred to as HKR. The motivation behind the model is an idea of long standing: consumers are antagonized by unexpected price hikes, and this may cause firms to be cautious when adapting prices to demand or cost shocks. (See Hall and Hitch (1939), Okun (1981) and Kahneman, Knetch and Thaler (1986).) In other words, consumers' distaste for unpleasant price surprises acts as a "menu cost" that deters firms from changing prices in response to exogenous shocks. Similar claims have been made in relation to wage rigidity in labor markets (see Fehr, Goette and Zehnder (2009)).[1]

The HKR model formalizes this idea. The distaste for unexpected price hikes is viewed as a manifestation of loss aversion (as in Kahneman and Tversky (1979)): unpleasant price surprises are perceived as a loss relative to the price the consumer expected, which acts as his reference point. HK apply the model of loss aversion preferences with expectations-driven reference points proposed by KR, and analyze

[1] Although this intuition is common and quite powerful, I am only aware of a single attempt to test it "in the field", Courty and Pagliero (2009). As it turns out, the evidence in this paper does not provide support for the antagonism hypothesis.

optimal monopoly pricing when consumers behave according to this model.[2]

I present a variation on the HKR model of optimal pricing when consumers are averse to unpleasant price surprises. I borrow the economic motivation as well as the main modelling idea from HKR, and do not claim any originality in this regard. However, I depart from the HKR model in several dimensions. Although the concept of loss aversion has been backed up by decades of experimental research, to turn it into a workable economic model the theorist has to make crucial modelling choices that themselves lack experimental support. Specifically, the theorist needs to address the following questions.

Which aspects of the market outcome are relevant for loss aversion?
In the HKR model, consumers display loss aversion both in the price dimension and in the consumption quantity dimension. The former captures the distaste for unpleasant price surprises, which is the focus of this paper. The latter, however, captures a distinct phenomenon, which is close both formally and psychologically to the well-known "endowment effect": the consumer experiences a disutility if his consumption quantity is lower than expected. The fact that both effects can be classified as instances of loss aversion attests to the power and generality of loss aversion as a theoretical construct. However, this does not change the fact that the two effects are distinct and not always equally applicable in a given market context. Therefore, I see no obvious reason to incorporate both of them into the same model. Therefore, *I assume loss aversion in the price dimension only.* As it happens, the two effects have contradictory pricing implications: distaste for price surprises leads to price rigidity, whereas the attachment effect may give the firm an incentive to randomize over prices. Thus, focusing on one of the effects while ignoring the other also makes it easier to obtain clear-cut results.

Does the consumer's expected action enter the specification of the reference point?
HKR assume that the consumer's reference point takes into account his own expectation of his own consumption decision. The consumer's decision is thus a "personal equilibrium": the action he chooses maximizes the consumer's reference-dependent utility given the reference point induced by the very same action. In our context, this would mean that the consumer's (stochastic) reference point is the price he expects to end up paying. In particular, if he expects not to buy the product, an unpleasant

[2]Heidhues and Kőszegi (2008) and Karle and Peitz (2008) extend this analysis to oligopolistic settings. For a different approach to modeling optimal pricing when consumers exhibit reference dependence, see Fibich et al. (2007).

price surprise does not generate a loss. In contrast, the model I present here assumes that *the reference point is the price the consumer expects the firm to charge*. Therefore, the consumer's expected action is irrelevant for the specification of the reference point, such that all personal-equilibrium considerations can be ignored.

How should we "sum over" multiple reference points?

The original specification of Prospect Theory assumed a single reference point. It is not clear how one should extend the model when there are multiple candidates for a reference point. A market environment with stochastic prices naturally generates a big set of possible outcomes that can act as reference points. HKR assume that the decision maker "sums over" them as follows. He computes his reference-dependent expected utility from a given action for any possible reference point r, and then integrates over all values of r to obtain his evaluation of the action. This procedure presumes that the consumer is aware of all possible reference points and allows each of them to influence his evaluation of each action. I believe, however, that reference points are powerful when they are salient; the greater the number of possible reference points the decision maker is aware of, the lower the likelihood that any of them will exert any influence on the consumer. Therefore, I assume that *the consumer has a single reference point in mind, but this reference point is drawn from the price distribution that characterizes the market*. In other words, while HKR "sum over" reference points across states within an individual consumers, I essentially sum the demands of different consumers with different reference points.

The reason for my first two departures from HKR is clear: one can fruitfully analyze consumers' aversion to unpleasant price surprises and its implications for price rigidity, without entering the complications that arise when we (1) introduce loss aversion into the consumer's evaluation of other aspects of the market outcome, and (2) allow the consumer's expectation of his own consumption decision to enter the specification of the reference point. In this sense, the model in this paper is a simplification of HKR, which is therefore useful for pedagogical purposes and for certain applications.

My third departure from HKR is a modification of their model of reference-point formation. Apart from the naturalness of assuming that consumers have a single reference point in mind, "sampling-based" reference-point formation has the merit that the resulting model of consumer choice treats the reference point as if it were a "consumer type". To obtain aggregate consumer demand, we simply integrate over all possible reference points, just as we integrate over all possible "standard" preference types. The difference is that the distribution of reference points, unlike the distribution of preference types, is endogenously determined by the firm's strategy.

I reproduce the main insights in HKR. Optimal prices are rigid in two respects. First, the price range is cramped relative to the no-loss-aversion benchmark: markups are higher in low-cost states and lower in high-cost states. Second, prices can be sticky in the sense of not responding at all to small cost shocks (I devote a lot less space to this effect than HKR). I then proceed to demonstrate the usefulness of the reformulated model with a pair of new results. First, I show that the expected price that firms charge is lower than in the benchmark without loss aversion.[3] Second, I demonstrate that the price rigidity effect is stronger in some sense when shocks are in demand rather than in costs.

The rest of the paper is structured as follows. In Section 2 I present the model. Section 3 provides a partial characterization of optimal pricing strategies and demonstrates price rigidity effects. Sections 4 and 5 are devoted to the new results: the impact of loss aversion on expected prices, and the difference between cost and demand shocks. Section 6 offers a few concluding remarks about the nature of the "cover version" exercise.

2 The Model

A monopolistic firm sells a single unit of a product to a continuum of measure one of consumers. The firm's marginal cost c is distributed uniformly over some finite set of possible values C. Denote $|C| = m > 1$. Let c_h and c_l denote the highest and lowest cost values in C, $1 > c_h > c_l > 0$. Let $\bar{c}$ denote the average cost in C. The firm commits to a deterministic pricing strategy $P : C \to \mathbb{R}$, where $P(c)$ is the price the monopolist charges when the marginal cost is c. Note that because of the randomness of the firm's marginal cost, its pricing strategy induces a probability measure μ_P over stated prices, where

$$\mu_P(x) = \frac{|\{c \in C \mid P(c) = x\}|}{m}$$

The consumers' choice model is as follows. Each consumer first draws a cost state c^e from the uniform distribution over C, and sets his reference price to be $p^e = P(c^e)$. Thus, the reference point is essentially drawn from μ_P. The consumer buys the product if and only if the actual price p satisfies $p \leq u - L(p, p^e)$, where: (i) u is the consumer's

[3] In their model of oligopoly pricing in the presence of loss averse consumers, Heidhues and Kőszegi (2008) report the following, related result, without formally deriving it: when loss aversion impacts the price dimension alone, it lowers the equilibrium price.

"raw" willingness to pay for the product, and (ii) L is a loss function given by

$$L(p, p^e) = \max[0, \lambda \cdot (p - p^e)] \tag{1}$$

where $\lambda > 0$. Assume that u is distributed uniformly over $[0, 1]$, independently of the reference point.

The piece-wise linear functional form of L has become "canonical" in the relevant literature, and is also employed by HKR. It captures the loss-aversion aspect of consumer behavior: consumers react to unpleasant price surprises with a reduced willingness to pay, while pleasant price surprises do not change their willingness to pay. The parameter λ captures the magnitude of consumer loss aversion.

The assumption that p^e is randomly drawn from μ_P requires justification. Clearly, there are many other ways to model the formation of the reference points. For instance, one could assume that the expected price according to μ_P is the consumer's reference point. The interpretation of "sampling-based" reference point formation is that the consumer creates his expectations on the basis of a random past market experience (his own or via word of mouth).

"Sampling-based" reference point formation implies that consumers effectively differ in two dimensions. First, they have different "raw" willingness to pay u for the product. Second, they have different market experiences that lead to different reference prices. Thus, the "sampling-based" formation of reference points enriches our conception of a consumer's "type"; moreover, it means that the firm can influence the distribution of consumer types, through its pricing strategy.

The monopolist's maximization problem can be written as a discrete optimal control problem. The firm chooses the function P to maximize

$$\Pi(P) = \frac{1}{m^2} \cdot \sum_{c} \sum_{c^e} [P(c) - c] \cdot \max[0, 1 - P(c) - L(P(c), P(c^e))] \tag{2}$$

To see why this is the objective function, note that for each realization of the marginal cost c, the firm is uncertain about the consumer's willingness to pay. Since his "raw" willingness to pay u is distributed uniformly over $[0, 1]$, in the absence of loss aversion (i.e., if L always takes the value zero) the probability that the consumer buys the product at a price $P(c)$ is $1 - P(c)$. However, in the presence of loss aversion, the probability that the consumer buys the product at $P(c)$ is $\max[0, 1 - P(c) - L(P(c), P(c^e))]$. Since the reference price $P(c^e)$ is drawn from μ_P, we need to sum over all possible values of c^e.

When L always gets the value zero, the optimal pricing strategy is

$$P^0(c) = \frac{1+c}{2} \tag{3}$$

for every $c \in C$. In comparison, if the firm were restricted to charging a constant price for all cost values, the optimal price would be

$$\bar{p} = \frac{1+\bar{c}}{2} \tag{4}$$

In the presence of loss aversion, consumer demand is a function of the entire price distribution induced by the firm's strategy. Specifically, given a pricing strategy P, the probability that the consumer buys the firm's product at a price p is

$$D_P(p) = \frac{1}{m} \sum_{c^e} \max[0, 1 - p - L(p, P(c^e))] \tag{5}$$

Thus, when the firm changes its price in one cost state, this affects (probabilistically) the consumer's reference point, hence consumer demand at other prices. In this sense, local prices changes have a global effect on consumer demand. When we hold P fixed, D_P decreases with p, as usual. Note that as long as $P(c) < 1$ for all c, $D_P[P(c)] > 0$ for all c. The reason is simple: when $P(c) < 1$, the consumer's "raw" willingness to pay u exceeds $P(c)$ with positive probability, while and the reference price is at least $P(c)$ with positive probability, such that $1 - P(c) - L(P(c), P(c^e)) > 0$ with positive probability.

Comment: Static vs. dynamic models

According to a literal interpretation of the model, the firm commits ex-ante to its pricing strategy. This is admittedly unrealistic. A less literal, and more interesting application is that the pricing strategy is induced by the firm's long-run behavior in a larger model, in which a patient, long-lived firm faces a sequence of short-lived consumers. In each period the firm makes a "spot" pricing decision in response to an *i.i.d* cost realization, taking into account the effect of the current price on future consumers' reference points. Providing a formal justification for this interpretation is outside the scope of the present paper.

Comment: Generalizing the model

The comparison conducted in the previous sub-section fails to do full justice to HKR, in the sense that while my model of consumer behavior has been tailored to the specific application of monopoly pricing, the original model due to Kőszegi and Rabin

(2006) provides a general "recipe" for modeling the behavior of loss averse agents with expectation-based reference points, which can be applied to any economic model in which the consequences that agents face are in $\mathbb{R}^2$ (and by simple extension, any Euclidean space). The behavioral model in this "cover version" of HKR can be similarly generalized. However, developing this theme is outside the paper's scope.

3 Reduced Price Variability

In this section I begin characterizing optimal pricing strategies. First, note that an optimal strategy necessarily exists. The reason is as follows. As we shall see below, it entails no loss of generality to assume that $P(c) \in [0,1]$ for every c. Thus, the domain of pricing strategies is $[0,1]^m$, a convex and compact set. By continuity of L, Π is a continuous function over this domain. Therefore, an optimum must exist.

The main result in this section is preceded by a pair of lemmas.

Lemma 1 *Let P be an optimal pricing strategy. Then, for every $c \in C$, $P(c) < 1$ and consumer demand (given by* (5)*) is strictly positive at $P(c)$.*

Proof. Let P be an optimal pricing strategy. Let us first show that $P(c) < 1$ for all $c \in C$. Assume, contrary to this claim, that $P(c^*) \geq 1$ for some $c^* \in C$. Then, consumer demand is zero at $P(c^*)$. Suppose that the firm deviates to a pricing strategy P' such that $P'(c^*) = 1 - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small, and $P'(c) = P(c)$ for all $c \neq c^*$. Following the deviation, consumer demand at $P'(c^*)$ is strictly positive, because $1 - P'(c^*) - L(P'(c^*), P'(c^*)) = 1 - P'(c^*) = \varepsilon > 0$. Let us now show that for any $c \neq c^*$, consumer demand at $P'(c)$ is the same as prior to the deviation, by going over all possible realizations of the reference price. Recall that $P'(c) = P(c)$. By definition, when $c^e \neq c^*$, $P'(c^e) = P(c^e)$, and therefore demand conditional on the reference price is unchanged. Now suppose that $c^e = c^*$. If $P(c) \geq 1$, then $1 - P'(c) = 1 - P(c) \leq 0$, hence demand is zero conditional on the reference price, both before and after the deviation. If, on the other hand, $P(c) < 1$, then $P(c) < P(c^*)$, and - if ε is sufficiently small - $P'(c) < P'(c^*)$, in which case $L(P'(c), P'(c^*)) = L(P(c), P(c^*)) = 0$, hence demand conditional on the reference price is unchanged. It follows that the deviation strictly raises the firm's expected profit. ■

The intuition for the result that aggregate consumer demand is always strictly positive is simple: for every price realization, there is positive probability that an individual consumer's reference price is weakly higher. Thus, any optimal pricing strategy P has the property that $D_P[P(c)] > 0$ for all c. The next lemma relies on this result.

Lemma 2 *Every optimal pricing strategy P is weakly increasing in c and satisfies $P(c) \geq c$ for every $c \in C$.*

Proof. Let P be an optimal pricing strategy. Suppose that $c_1 > c_2$ and yet $P(c_2) > P(c_1)$. Denote $P(c_1) = p_1$ and $P(c_2) = p_2$. Suppose that the firm switches to a pricing strategy P' that coincides with P for all $c \neq c_1, c_2$, and $P'(c_1) = p_2$, $P'(c_2) = p_1$. The change in the firm's objective function as a result of the deviation is

$$\frac{1}{m}\sum_c [(P'(c) - c) \cdot D_{P'}(P'(c)) - (P(c) - c) \cdot D_P(P(c))]$$

The deviation has the property that it does not alter the induced price distribution. That is, $\mu_P = \mu_{P'}$. Therefore, the deviation does not change consumer demand: $D_{P'}(x) = D_P(x)$ for every price x. Accordingly, let us omit the subscript of the demand function. We can thus rewrite the above expression as follows:

$$\frac{1}{m}\sum_c \{[P'(c) - c]D(P'(c)) - [P(c) - c]D(P(c))\}$$

Since P and P' coincide over $c \neq c_1, c_2$, this expression is strictly positive if and only if

$$[p_2 - c_1]D(p_2) - [p_1 - c_1]D(p_1) + [p_1 - c_2]D(p_1) - [p_2 - c_2]D(p_2) > 0$$

which simplifies into

$$(c_2 - c_1)(D(p_2) - D(p_1)) > 0$$

Recall that by assumption, $c_1 > c_2$ and $p_2 > p_1$. Since D is strictly decreasing in the range in which it is strictly positive, and since we have established in the previous lemma that $D(P(c)) > 0$ for all $c \in C$, the inequality holds. Therefore, the deviation to P' is strictly profitable.

Let $C^* = \{c \in C \mid P(c) < c\}$. Suppose that C^* is non-empty. Consider a deviation to a pricing strategy P' that satisfies $P'(c) = c$ for all $c \in C^*$ and $P'(c) = P(c)$ for all $c \notin C^*$. For every $c \in C^*$, the firm's profit conditional on being chosen ceases to be strictly negative as a result of the deviation. For every $c \notin C^*$, the deviation weakly lowers the loss aversion term because it raises the consumers' reference price with positive probability. Since we have established that consumer demand is strictly positive at all cost states, the deviation is strictly profitable. Therefore, C^* must be empty. ■

In the absence of loss aversion, this result follows trivially from (3). In the presence of loss aversion, it is a non-trivial result because in general, the firm's pricing strategy influences consumer demand. However, when the firm deviates to a pricing strategy that does not change the overall price distribution, consumer demand is unaffected. The proof is based on this type of deviation.

We are now ready to prove our first main result. When the consumer is loss averse, the firm raises the price at the lowest level of marginal cost and lowers the price at the highest level of marginal cost, relative to the benchmark with no loss aversion. Thus, optimal pricing strategies exhibit *reduced price variability*, compared with the benchmark without loss aversion. The firm does not want the consumer to experience large losses that will reduce his willingness to pay, and therefore shrinks the price range.

Proposition 1 *Let P be an optimal pricing strategy P. Then:*

$$P^0(c^l) \leq P(c^l) \leq P(c^h) \leq P^0(c^h)$$

Proof. Let P be an optimal pricing strategy. By Lemma 2, P is weakly increasing and satisfies $P(c) \geq c$ for every $c \in C$, hence $P(c^h) \geq P(c^l) \geq c^l$ and $P(c^h) \geq c^h$.

(i) $P^0(c^l) \leq P(c^l)$. Recall that $P^0(c^l) = \frac{1}{2}(1 + c^l)$. In the absence of loss aversion, the hump shape of the firm's objective function implies that for every $c \geq c^l$, the price $P^0(c^l)$ yields a higher profit than any $p' < P^0(c^l)$. Define c^0 to be the highest cost c for which $P(c) < \frac{1}{2}(1 + c^l)$. Since P is weakly increasing, $P(c) < \frac{1}{2}(1 + c^l)$ for all $c \leq c^0$. Suppose that the firm deviates to a pricing strategy P' that satisfies $P'(c) = \frac{1}{2}(1 + c^l)$ for $c \leq c^0$ and coincides with P for $c > c^0$. The "bare" profit excluding the loss aversion term goes up. As to the loss aversion component, because P' is flat over $c \leq c^0$, $L(P'(c_1), P'(c_2)) = 0$ whenever $c_1, c_2 \leq c^0$. Moreover, because $P'(c) > P(c)$ for $c \leq c^0$ and $P'(c) = P(c)$ for $c > c^0$, $L(P'(c_1), P'(c_2)) \leq L(P(c_1), P(c_2))$ whenever $c_2 \leq c^0 < c_1$. Since P and P' coincide at all $c > c^0$, $L(P'(c_1), P'(c_2)) = L(P(c_1), P(c_2))$ when $c_1, c_2 > c^0$. Finally, when $c_1 \leq c^0 < c_2$, $L(P'(c_1), P'(c_2)) = 0$ because by the definitions of c^0 and P', $P'(c_2) > P'(c_1)$. It follows that the deviation is profitable.

(ii) $P(c^h) \leq P^0(c^h)$. Consider two cases. First, suppose that P is flat - i.e., $P(c^l) = P(c^h) = \bar{p}$. We have seen that in this case, the optimal price $\bar{p}$ is given by (4),

which is strictly below $\frac{1}{2}(1+c^h)$. Second, suppose that $P(c^l) < P(c^h) = p^h$. Let C^* be the set of cost values c for which $P(c) = P(c^h) = p^h$. Since P is weakly increasing in c, there exists $c^* \in C$ such that $C^* = \{c \in C \mid c > c^*\}$. By (5), consumer demand at p given P is strictly lower than $1-p$ for every $p > p^l$. Assume that $p^h > \frac{1}{2}(1+c^h)$. Suppose that the firm switches to a pricing strategy P' such that $P'(c) = P(c) - \varepsilon$ for all $c \in C^*$ and $P'(c) = P(c)$ for all $c \notin C^*$. If $\varepsilon > 0$ is sufficiently small, $P'(c) > P'(c^e)$ for every $c \in C^*$, $c^e \notin C^*$. By the hump shape of the firm's profit function in the absence of loss aversion, this deviation strictly raises this "bare" profit in all states $c \in C^*$. In addition, the deviation weakly lowers the loss aversion term in those states, without changing the loss aversion term in the states outside C^*. Therefore, the deviation is strictly profitable. ■

Can Proposition 1 be strengthened, such that the weak inequalities in the statement of the result are replaced with strict ones? The answer is negative. For instance, suppose that C consists of two cost values. It is easy to show in this case that when the loss aversion parameter λ is sufficiently large, one of the following strategies is optimal: the no-loss-aversion benchmark strategy given by (3), or the constant price given by (4). (Which of the two shall prevail depends on the cost values.) Thus, each one of the three inequalities in the statement of the proposition can be weak.

Price Stickiness

So far, I used the term "price rigidity" to describe a cramped range of prices compared with the benchmark pricing strategy P^0. However, when economists discuss price rigidity, they often have in mind a "stickiness" property - namely, lack of response to small shocks. The following result employs a two-state example to illustrate this property.

Proposition 2 *Let $C = \{c, c+2\varepsilon\}$. When $\varepsilon > 0$ is sufficiently small, the optimal pricing strategy is to charge a constant price $\bar{p} = \frac{1}{2}(1+c+\varepsilon)$.*

Proof. Let P be an optimal pricing strategy. Denote $P(c) = p_l$, $P(c+2\varepsilon) = p_h$. Assume that P exhibits price variation - i.e., $p_h > p_l$. If $1 - p_h - L(p_h, p_l) \leq 0$, the firm's expected profit is

$$\frac{1}{2}(p_l - c)(1-p_l) + \frac{1}{2}(p_h - c - 2\varepsilon) \cdot \frac{1}{2}(1-p_h)$$

As long as $\varepsilon < \frac{1}{4}$, this is strictly lower than $\frac{1}{4}(1-c-\varepsilon)^2$, which is the profit from the optimal constant price.

Now suppose that in optimum, $1 - p_h - L(p_h, p_l) > 0$. Then, the solution is characterized by first-order conditions:

$$\begin{aligned}\frac{\partial\Pi(p_l,p_h)}{\partial p_l} &= 1 + c - 2p_l + \frac{\lambda}{2}(p_h - c - 2\varepsilon) = 0 \\ \frac{\partial\Pi(p_l,p_h)}{\partial p_h} &= 1 + c + 2\varepsilon - 2p_h - \frac{\lambda}{2}(p_h - c - 2\varepsilon) - \frac{\lambda}{2}(p_h - p_l) = 0\end{aligned}$$

such that

$$2(p_h - p_l) = 2\varepsilon - \lambda(p_h - c - 2\varepsilon) - \frac{\lambda}{2}(p_h - p_l)$$

Since $p_h > p_l$, this equation implies the following inequality

$$\lambda \cdot (p_h - p_l) < 4\varepsilon - 2\lambda(p_h - c - 2\varepsilon)$$

By Proposition 1, $p_h > \frac{1}{4}(1 + c)$, hence

$$\lambda \cdot (p_h - p_l) < 4\varepsilon - 2\lambda(\frac{1}{4} + c - 2\varepsilon) \tag{6}$$

Since $\lambda > 0$, it is easy to see that if ε is sufficiently small, the R.H.S of (6) is negative, implying $p_h - p_l < 0$, a contradiction. ∎

Thus, when cost fluctuations are small, it is optimal for the firm to charge a fully rigid price that does not respond to the cost change. When C is larger and contains additional cost values, it is harder to check whether it is optimal to set $P(c) = P(c+2\varepsilon)$. The reason is that as the firm moves $P(c)$ and $P(c + 2\varepsilon)$ closer together, it affects the loss aversion term when the actual or reference price lies outside $\{P(c), P(c+2\varepsilon)\}$. For instance, slightly raising $P(c)$ lowers demand when $P(c)$ is the actual price and the reference price is $P(c')$ for some $c' < c$, but raises demand when $P(c)$ is the reference price and $P(c')$ is the actual price for some $c' > c + 2\varepsilon$.

Comment: Random pricing strategies

The assumption that the firm is restricted to deterministic pricing strategies is without loss of generality. That is, the firm would not have a incentive to employ random pricing strategies even if those were feasible. To see why, observe that the functional form of L given by (1) is a maximum of linear functions and therefore a convex function of (p, p^e). This in turn implies that Π is a concave function over the relevant domain $[0, 1]^{|C|}$. As a result, randomization cannot be strictly optimal.

The issue of random prices is related to the assumption that u is uniformly distributed. The primary motivation for this assumption is notational simplicity. However, the "swapping argument" in the proof of Lemma 2 relies on the uniformity assumption. In order to extend this argument to non-uniform distributions of u, one would have to assume that random pricing strategies are feasible - even though randomization is ultimately sub-optimal.[4]

4 Impact of Loss Aversion on the Average Price

What is the effect of loss aversion on the *expected* price that the monopolist charges in optimum? Recall that if the firm adopts a constant price $\bar{p}$, then $\bar{p} = \frac{1}{2}(1+\bar{c})$ - i.e., the expected price is exactly as in the no-loss-aversion benchmark. The question is how the expected price changes when there is some price variability. For simplicity, assume that loss aversion is not too strong, in the following sense:

$$\lambda < \frac{1-c^h}{c^h - c^l} \tag{7}$$

Given Proposition 1, this restriction ensures that $1 - P(c) - L(P(c), P(c^e)) > 0$ for all c, c^e under any optimal pricing strategy P. In other words, consumer demand is strictly positive conditional on any realization of actual and reference prices. The average price that the firm charges under P is simply

$$\frac{1}{m}\sum_{c \in C} P(c)$$

Proposition 3 *If an optimal pricing strategy is not a constant function, the average price it induces is strictly below* $\bar{p} = \frac{1}{2}(1+\bar{c})$.[5]

Proof. Let P be a non-constant optimal pricing strategy. Since P maximizes the firm's expected profit, it satisfies the following inequality for every alternative strategy Q:

$$\begin{aligned} &\sum_{c}\sum_{c^e}[P(c) - c] \cdot \max[0, 1 - P(c) - L(P(c), P(c^e))] \\ \geq &\sum_{c}\sum_{c^e}[Q(c) - c] \cdot \max[0, 1 - Q(c) - L(Q(c), Q(c^e))] \end{aligned}$$

[4] Heidhues and Kőszegi (2010) analyze the model of HK with homogenous consumers and no cost fluctuations, but -.unlike HK - allowing for price randomization. They show that the optimal pricing strategy combines rigidity with randomization. The latter effect is due to loss aversion in the consumption dimension.

[5] This proof follows a suggestion by Ariel Rubinstein and supplants a former proof based on first-order conditions.

Define $Q(c) \equiv P(c) - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small. Then, since by definition L is only a function of the difference between the actual and expected price, the following inequality holds:

$$\sum_c \sum_{c^e} [P(c) - c] \cdot \max[0, 1 - P(c) - L(P(c), P(c^e))]$$
$$\geq \sum_c \sum_{c^e} [P(c) - \varepsilon - c] \cdot \max[0, 1 - P(c) + \varepsilon - L(P(c), P(c^e))]$$

Condition (7) ensures that $1 - P(c) + \varepsilon - L(P(c), P(c^e)) > 0$ for all c, c^e. It follows that we can rewrite the above inequality as follows:

$$\sum_c \sum_{c^e} \{[P(c)-c] \cdot [1-P(c)-L(P(c), P(c^e))] - [P(c)-\varepsilon-c] \cdot [1-P(c)+\varepsilon-L(P(c), P(c^e))]\} \geq 0$$

This inequality is simplified into

$$2 \sum_c \sum_{c^e} P(c) \leq \sum_c \sum_{c^e} [1 + c + \varepsilon - L(P(c), P(c^e))]$$

Since P is not a constant function, $\sum_c \sum_{c^e} L(P(c), P(c^e)) > 0$. Therefore, if ε is sufficiently close to zero, we can write

$$2 \sum_c \sum_{c^e} P(c) < \sum_c \sum_{c^e} (1 + c)$$

That is:

$$2m \sum_c P(c) < m \sum_c (1 + c)$$

which immediately implies the result. ■

The rough intuition for this result is as follows. Since loss aversion diminishes willingness to pay, it effectively results in a downward vertical shift of the linear demand function that the firm faces. These forces impel the firm to lower its price on average.

Note that the expected price does not decrease monotonically with the intensity of loss aversion. Proposition 3 establishes that when λ is relatively small, the expected price is strictly lower than in the no-loss-aversion benchmark. However, we already observed in Section 3 that when λ is sufficiently large (such that in particular, condition (7) is violated), the firm may revert to the no-loss-aversion pricing strategy or switch to the optimal constant price, both of which induce an average price of $\frac{1}{2}(1 + \bar{c})$.

5 Cost vs. Demand Fluctuations

Throughout the paper, I have followed HKR by assuming that fluctuations occur in the cost dimension only. It is straightforward to extend the model by introducing aggregate demand fluctuations as well, and allowing the firm to condition its price on both cost and demand shocks. Let Ω be a finite set consisting of m *states*. The firm's prior over Ω is uniform. Each state $\omega \in \Omega$ is characterized by a cost-demand pair of parameters (c_ω, v_ω), where $v_\omega > c_\psi$ for all $\omega, \psi \in \Omega$. In state ω, the measure of the consumer population is v_ω, and the consumers' "raw" willingness to pay is uniformly drawn from the interval $[0, v_\omega]$. Thus, an increase in v corresponds to an upward shift in the demand function faced by the monopolist. Let $\bar{c}$ and $\bar{v}$ denote the average values of c and v across all states.

A pricing strategy is a function $P : \Omega \to \mathbb{R}$. The firm's objective function is:

$$\Pi(P) = \frac{1}{m^2} \cdot \sum_{\omega \in \Omega} \sum_{\psi \in \Omega} [P(\omega) - c_\omega] \cdot \max[0, v_\omega - P(\omega) - L(P(\omega), P(\psi))]$$

In the benchmark without loss aversion (i.e., $L \equiv 0$), the firm's optimal pricing strategy is

$$P^0(\omega) = \frac{v_\omega + c_\omega}{2}$$

The optimal constant price is

$$\bar{p} = \frac{\bar{v} + \bar{c}}{2}$$

Thus, in the absence of loss aversion, the optimal pricing strategy treats demand and cost shocks symmetrically. This raises the question of whether this equal treatment property carries over when consumers display loss aversion. In this section, I show that the answer is negative. Moreover, the firm's incentive to employ a rigid pricing strategy is stronger in some sense when fluctuations occur in the demand dimension.

5.1 A Two-State Example

Assume that there are two states of nature, l and h. Let $v_h \geq v_l$ and $c_h \geq c_l$, with at least one strict inequality. A pricing strategy is thus represented by a pair of prices (p_l, p_h), where $p_\omega = P(c_\omega, v_\omega)$. It can be shown (in the manner of Lemma 2) that any optimal pricing strategy satisfies $p_h \geq p_l$. Since $L(p, p^e) = 0$ whenever $p \leq p^e$, the objective function can be simplified into:

$$(p_l - c_l) \max(0, v_l - p_l) + (p_h - c_h)[\frac{1}{2} \max(0, v_h - p_h) + \frac{1}{2} \cdot \max(0, v_h - p_h - L(p_h, p_l))]$$

When λ is sufficiently small, consumer demand is guaranteed to be strictly positive for all realizations of reference and actual prices, such that the objective function is further simplified into

$$(p_l - c_l)(v_l - p_l) + (p_h - c_h)[v_h - p_h - \frac{1}{2}L(p_h, p_l)]$$

In addition, the solution to the firm's maximization problem, denoted (p_l^*, p_h^*), is given by first-order conditions:

$$\begin{aligned}
\frac{\partial \Pi(p_l^*, p_h^*)}{\partial p_l} &= v_l + c_l - 2p_l^* + \frac{\lambda}{2}(p_h^* - c_h) = 0 \\
\frac{\partial \Pi(p_l^*, p_h^*)}{\partial p_h} &= v_h + c_h - 2p_h^* - \frac{\lambda}{2}(p_h^* - c_h) - \frac{\lambda}{2}(p_h^* - p_l^*) = 0
\end{aligned}$$

leading to the solution:

$$\begin{aligned}
p_l^* &= \frac{1}{8\lambda - \lambda^2 + 16}[(8 + 4\lambda)(v_l + c_l) + 2\lambda(v_h + c_h) - c_h(4\lambda + \lambda^2)] \\
p_h^* &= \frac{1}{8\lambda - \lambda^2 + 16}[8(v_h + c_h) + 2\lambda(v_l + c_l) + c_h(4\lambda - \lambda^2)\}]
\end{aligned}$$

which satisfies $p_h^* > p_l^*$.

These expressions allow us to compare two environments, A and B, that are identical in every respect except that in environment A fluctuations are in costs whereas in environment B fluctuations are in demand:

$$\begin{array}{rcl}
v_l^A = \bar{v} & \quad & v_l^B = \bar{v} - \varepsilon \\
v_h^A = \bar{v} & & v_h^B = \bar{v} + \varepsilon \\
c_l^A = \bar{c} - \varepsilon & & c_l^B = \bar{c} \\
c_h^A = \bar{c} + \varepsilon & & c_h^B = \bar{c}
\end{array}$$

Both environments share the same expected values of c and v. Moreover, $v_l^A + c_l^A = v_l^B + c_l^B = \bar{v} + \bar{c} - \varepsilon$ and $v_h^A + c_h^A = v_h^B + c_h^B = \bar{v} + \bar{c} + \varepsilon$. When $\lambda > 0$, in environment B, p_h^* is lower and p_l^* is higher than in environment A. In other words, the optimal pricing strategy displays greater rigidity in environment B.

The intuition for this result is as follows. The loss aversion term is contributed by the event in which the consumer's reference price is p_l and the actual price he faces is p_h. When the firm contemplates raising p_l or lowering p_h, it does so to curb the loss aversion term. The mark-up in state h, $p_h - c_h$, is higher when fluctuations are in demand rather than in costs. Therefore, the firm's gain from curbing the loss aversion term and thus raising expected demand in state h is larger in environment B. As a result, the firm's incentive to narrow the price range is stronger in the case of demand shocks.

Comment. There is a common intuition that consumer antagonism to unexpected price hikes is stronger when these follow a surge in demand rather than in costs. Kahneman et al. (1986) provide experimental support for this intuition. The fact that the results in this section are consistent with this pattern is a mere coincidence. In the experiments of Kahneman et al. (1986), subjects are outraged by price hikes due to a surge in demand because they perceive them as unfair and exploitative, compared with a price increase in response to a cost shock. These intuitive judgments rely on an element of social preferences that is entirely absent from both the HKR model and its current "cover version".

5.2 Preference for a Fully Rigid Price

Let us now turn back to the case of an arbitrary number m of states, and compare two environments, A and B, which are related to each other as follows. First, the state space in both environments has the same cardinality m. Second, $v_\omega^A + c_\omega^A = v_\omega^B + c_\omega^B$ for every ω. Third, $\bar{c}^A = \bar{c}^B = \bar{c}$ and $\bar{v}^A = \bar{v}^B = \bar{v}$. Finally, $v_\omega^A = \bar{v}^A$ and $c_\omega^B = \bar{c}^B$ across all ω. Thus, the two environments are identical in every respect, except that in environment A the fluctuations are in costs whereas in environment B the fluctuations are in demand. In the absence of loss aversion, the two environments generate the same optimal pricing strategy.

Our next result reveals a sense in which the incentive to employ rigid pricing strategies is stronger under demand fluctuations. We will say that a pricing strategy P is *regular* if it satisfies two properties: *(i)* it weakly increases with c and v; *(ii)* it induces strictly positive consumer demand for all realizations of actual and reference prices.

Proposition 4 *If the firm prefers the optimal constant price to a regular pricing strategy P in environment A, then it must have the same preference in environment B.*

Proof. The firm's expected profit from the optimal constant price $\bar{p}$ is the same under both environments. Thus, we only need to show that the firm's expected profit from an arbitrary regular pricing strategy P is higher in environment A than in environment B. Since P is regular, we can rewrite its expected profit in each environment (omitting the multiplicative term $1/m^2$) as follows:

$$\begin{aligned}
\Pi(P) &= \sum_\omega [P(\omega) - c_\omega][v_\omega - P(\omega)] - \sum_\omega (P(\omega) - c_\omega) \sum_\psi L(P(\omega), P(\psi)) \\
&= \sum_\omega P(\omega)[v_\omega + c_\omega - P(\omega)] - \sum_\omega c_\omega v_\omega \\
&\quad - \sum_\omega \sum_\psi P(\omega) \cdot L(P(\omega), P(\psi)) + \sum_\omega \sum_\psi c_\omega \cdot L(P(\omega), P(\psi))
\end{aligned}$$

The first term in the final expression for $\Pi(P)$ is identical for both environments, by the assumption that $v_\omega^A + c_\omega^A = v_\omega^B + c_\omega^B$ for every ω. The second term is identical for both environments, by the assumption that $\bar{c}^A = \bar{c}^B = \bar{c}$ and $\bar{v}^A = \bar{v}^B = \bar{v}$, coupled with the assumption that $v_\omega^A = \bar{v}^A$ and $c_\omega^B = \bar{c}^B$ across all ω. The third term is identical for both environments because it is only a function of the pricing function and not of underlying cost and demand parameters. It therefore remains to compare the fourth term under both environments. Let us rewrite this fourth term as follows:

$$\sum_\omega c_\omega L_\omega^* \tag{8}$$

where

$$L_\omega^* = \sum_\psi L(P(\omega), P(\psi))$$

Order the states in Ω according to their $v_\omega + c_\omega$ (which is the same in both environments A and B). By assumption, P is weakly increasing in $v_\omega + c_\omega$. Therefore, L_ω^* is also increasing in $v_\omega + c_\omega$. By assumption, both environments A and B share the same $\sum_\omega c_\omega$, yet $c_\omega^B = \bar{c}$ for all ω, whereas c_ω^A increases with $v_\omega + c_\omega$. It follows that expression (8) is higher in environment A. Therefore, the firm's expected profit from P is higher in environment A. ■

The intuition for this result - as in the two-state example - is that when the firm contemplates a small change in the price in some state, it is mindful of the implications of this price change for the loss aversion term in other states associated with higher prices. When fluctuations are in demand, the mark-up in those high-price states is higher than when fluctuations are in cost, and therefore the incentive to shrink the gap between the high and low prices is stronger.

6 Conclusion

This paper has presented a re-modelling exercise, referred to as a "cover version" of the HKR model. Hopefully, the variation analyzed here contributes a different way of looking at the notion of reference-dependent consumer preferences. Its usefulness was demonstrated with a pair of novel results, concerning the impact of loss aversion on expected price and the difference between cost and demand fluctuations. An important ingredient in the HKR model, namely the notion of personal equilibrium, has been suppressed by the assumption that the consumer's reference point does not depend on his own expected action. Combining such a notion with the "sampling-based" method of aggregating reference points is left for future research. See Spiegler (2011) for a proposal for such an extended model.

Although the model presented here is a simplification of HKR (and as such, potentially useful pedagogically as well as for certain applications), this has not been the main point of the re-modelling exercise. The current norm in our profession is that when a theorist first proposes a way to model a certain phenomenon, this model becomes a benchmark for future analyses of the phenomenon. Subsequent models are expected to differentiate themselves, horizontally (dealing with distinct phenomena) or vertically (extending the original model). It is as we provide the original model with some kind of "patent protection". In this paper, I wanted to pursue a different approach, which continues to give the original model the full credit it deserves for being the first to deal with a phenomenon, yet at the same time removes the "patent protection" and develops an alternative model of the phenomenon as if the original model did not exist. I believe that just as lifting excessive patent protection can sometimes foster technological innovation, so giving room to "cover versions" of the type pursued here can benefit the development of economic theory.

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