Competitive Screening in Credit Markets with Adverse Selection: A "Cover Version" of Bester's Model

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Abstract

This note presents a simple model of competitive screening in a credit market with adverse selection, where firms use interest and collateral as instruments for screening borrowers' risk type. While the basic ideas appear in Bester (1985,1987), this version of the model is far more useful pedagogically, also (arguably) when compared with models that are more conventionally employed for this purpose.

1 Introduction

This note analyzes an example a competitive market with adverse selection, which is in my opinion superior pedagogically to conventionally used models, and therefore comfortably fits an intermediate microeconomics course. Familiar pedagogical expositions of competitive screening models use the pioneering model of insurance markets due to Rothschild and Stiglitz (1976, RS henceforth), or the labor-market example adopted by Green, Mas-collel and Whinston (1995, GMW henceforth), where contracts are defined by wage and training. In my years as a teacher of intermediate microeconomics, I found both examples wanting. The RS model involves non-linear utility functions because the model crucially relies on risk aversion. Therefore, analytic solutions are cumbersome and the instructor typically resorts to diagrammatic exposition, which then makes it hard to compose problem sets and exam questions.

Furthermore - although this may seem strange to a professional economist - it is not so easy to find clean real-life examples of insurance markets in which adverse selection is an obvious phenomenon. If the context is car insurance, then the assumption that drivers know their driving risks better than insurance companies is controversial. For instance, a young male driver is likely to overestimate his driving skills, and the insurance firm - knowing his gender and age group - is perhaps more likely to have a good assessment. Indeed, Chiappori and Salanie (2000) have called into question the empirical validity of this assumption in the context of car insurance. As to health insurance, in many countries private health insurance is not dominant. In a single-payer system, the insurer is also a provider of medical services, and in this case it is harder to motivate the assumption that a patient's medical condition is his private information. Although the student should be able to imagine a health insurance market that fits the model, this requires effort and special pleading. It is undoubtedly great as an application of the model, but perhaps not ideal as a first illustration.

Finally, the labor-market example of GMW is not completely natural, and therefore harder to motivate in front of a skeptical audience. Explicit contracts that condition wage on training are rare, and so the student is invited to interpret the model more abstractly as a norm that governs the worker's expectations. There is nothing wrong with that, of course, but again, this is probably not ideal as a first example. When presenting such an important subject, one would like to have an example that is as easy to motivate and analyze as possible.

In this note I present such an example of a competitive screening model. I examine a credit market, where lenders use interest and collateral as instruments to screen the riskiness of borrowers. The basic modeling idea appears in Bester (1985,1987). However, Bester's intention was not pedagogical, and therefore his model is not presented in a way that is ideal for a textbook or classroom presentation. I believe that the present version is a significant improvement in this regard. Specifically, it has two merits that address the above criticisms. First, the model is entirely linear, hence the instructor can easily use both diagrams and analytical characterization to present the model and devise problem sets and exam questions. Second, the story is compelling and does not require special pleading. It is particularly attractive for students, given the intense attention to credit-market failures (and the role of collateral in particular) in the wake of the 2007/8 financial crisis. In order to make this note as pedagogically useful as possible, I provide detailed proofs of the results, even though they are based on entirely familiar| ideas.

Although the subject matter of this note is highly conventional, its genre is not. Rewriting existing models in a way that makes them more suitable for pedagogical exposition is part of doing economic theory. Theorists do this all the time, but they generally keep the outputs of this activity to themselves, in the form of unpublished lecture notes, possibly because current publication norms constrain the dissemination of such outputs - but see Spiegler (2012) for a recent exception to this norm. Hopefully, this modest note will encourage other theorists to make pedagogically useful "cover versions" of existing models publicly available in refereed journals.

2 A Model

A borrower requires a loan of size 1 in order to finance a project, the value of which is also 1. A loan contract is a pair of numbers (r, x), $r \geq 0$ and $x \in [0, 1]$, where r is the interest on the loan and x is the amount of collateral that the borrower is required to deposit when he receives the loan. Note that the fruits of the borrower's project cannot serve as collateral. For each borrower, there is a probability q - referred to as the borrower's credit risk - that he will become insolvent and thus unable to repay back the loan (either the principal or the interest). In this case, the collateral that the borrower deposited in advance can be seized by the lender. However, there is an inefficiency in realizing the collateral, such that the value to the lender of a collateral of size x is bx, where $b \in (0, 1)$. Assume that there are two borrower types in the population, 1 and 2, with credit risk q_1 and q_2, respectively, where $0 < q_1 < q_2 < \frac{1}{2}$. The fraction of each type in the population of borrowers is 50%. We will assume that the credit market is competitive, in the sense that there is a large number of profit-maximizing lenders who can extend any loan contract. We will also assume that all agents in this market are risk-neutral.

3 Complete-Information Benchmark

Suppose that a borrower's credit risk is public information. Then, the credit market is segmented into two separate sub-markets, one for each type. Consider an arbitrary type and denote his credit risk by q. Then, his expected utility from a loan contract (r, x) is

$$1 - r(1 - q) - qx \tag{1}$$

because the value of financing the project is 1, the net payment to the lender is r when the borrower is solvent (an event that occurs with probability $1 - q$), and the net payment to the lender is x when the borrower is insolvent (an event that occurs with probability q). If the borrower opts out, his payoff is 0 (because the project is not materialized).

The lender's expected profit from the same contract is

$$r(1-q) - q(1-bx) \tag{2}$$

because with probability $1-q$ the lender gets his money back plus interest, while with probability q, he loses the principal of the loan and can only reclaim the value of the collateral.

A competitive equilibrium in this complete-information environment is a contract (r, x) that satisfies two requirements: (i) it generates zero profits given borrowers' choice (between the contract and the outside option); (ii) there exists no new contract (r', x') that could enter the market and generate positive profits, given borrowers' choice (among the original contract, the new contract and the outside option).

Claim 1 *The competitive equilibrium contract is* $r^* = \frac{q}{1-q}$, $x^* = 0$.

Proof. First, we will show that (r^*, x^*) is a competitive equilibrium. Next, we will show that there exist no other equilibria.

Plugging (r^*, x^*) into (1), we see that the contract generates an expected utility of $1-q > 0$ for borrowers, and plugging (r^*, x^*) into (2), we see that the contract generates zero profits for lenders. If a new contract (r, x) enters the market, it generates positive profits if and only if

$$\begin{aligned} r(1-q) - q(1-bx) &> 0 \\ 1 - r(1-q) - qx &> 1-q \end{aligned}$$

These conditions cannot hold at the same time because $b < 1$. Therefore, (r^*, x^*) is an equilibrium.

Now suppose that there is another equilibrium contract (r, x). By the zero-profit condition, it must be the case that $x > 0$. Now consider a new contract $(r + \delta, x - \varepsilon)$ that enters the market, where $\varepsilon > 0$ is small, such that $x - \varepsilon > 0$. This contract will attract the borrowers and generate positive profits if and only if

$$\begin{aligned} \delta(1-q) - qb\varepsilon &> 0 \\ \delta(1-q) - q\varepsilon &< 0 \end{aligned}$$

We can set δ such that

$$\frac{bq\varepsilon}{1-q} < \delta < \frac{q\varepsilon}{1-q}$$

because $b < 1$. Therefore, the original contract (r, x) cannot be an equilibrium. ■

Thus, competitive equilibrium under complete information precludes the use of collateral. Contracts with collateral are Pareto-inefficient because they mean that with some probability, the borrower transfers to the lender something that is more valuable to the borrower (this feature is captured by the assumption that $b < 1$). Under complete information, competitive forces induce such an efficient outcome, while the interest rate is determined by the zero-profit condition.

4 Competitive Equilibrium under Asymmetric Information

Now suppose that a borrower's credit risk is his private information. In this case, lenders are unable to identify the type of borrower that takes a give loan, and therefore cannot explicitly target contract loans for a certain type of borrowers. When borrowers enter the market, they can choose from a set of available loan contracts. In addition, they can choose to opt out and get a payoff of zero. Denote the outside option by c_0. In this environment, a competitive equilibrium is a collection C of contracts of the form (r, x), such that: (i) each contract in C generates zero profits, given the way each type of borrower chooses from the set $C \cup \{c_0\}$; (ii) there exists no new contract $c \notin C$ that would generate positive profits, given the way each type of borrower chooses from the set $C \cup \{c_0, c\}$.

Let us analyze competitive equilibria in this model. Throughout the analysis, I will take it for granted that each type of borrower prefers every contract in C to the outside option c_0. It can be verified that our assumption that $q_2 < 1$ ensures that this is indeed the case. Let us begin with pooling equilibria. In a pooling equilibrium, all borrowers choose the same contract.

Claim 2 *There exists no pooling equilibrium.*

Proof. Assume that a pooling equilibrium exists. Let (r, x) be the equilibrium contract. Since it is selected by all types of borrowers, the expected profit that it generates is

$$(1 - \bar{q})r - \bar{q}(1 - bx)$$

where

$$\bar{q} = \frac{q_1 + q_2}{2}$$

is the average credit risk in the pool of borrowers who select the contract. We will show that a new contract (r', x') can enter and attract only low-risk borrowers and make a positive profit. In other words, the new contract will satisfy the following inequalities:

$$\begin{aligned}(1-q_1)r' + q_1 x' &< (1-q_1)r + q_1 x \\ (1-q_2)r' + q_2 x' &> (1-q_2)r + q_2 x \\ (1-q_1)r' - q_1(1-bx') &> 0\end{aligned}$$

The first inequality means that for type 1, expected payments are lower under the new contract. The second inequality means that for type 2, expected payments are lower under the original contract. The third inequality means that when only borrowers of type 1 select the new contract, it generates positive profits. Denote

$$\begin{aligned}x' &= x + \varepsilon \\ r' &= r - \delta\end{aligned}$$

That is, the new contract slightly raises the collateral requirement in return for a slightly lower interest. The first pair of inequalities can be rewritten as follows:

$$\frac{q_1}{1-q_1}\varepsilon < \delta < \frac{q_2}{1-q_2}\varepsilon$$

For every $\varepsilon > 0$ we can find $\delta > 0$ that satisfies these inequalities, because $q_1 < q_2$. The third inequality (namely, the positive-profit condition) can be rewritten as follows:

$$[(1-q_1)r - q_1(1-bx)] + [bq_1\varepsilon - \delta(1-q_1)] > 0$$

If ε and δ are small (and we can set them to be arbitrarily close to zero), the term in the second square brackets is small (that is, close to zero). As to the term in the first square brackets, recall that by the assumption that the contract (r, x) generates zero profits when all borrower types select it:

$$(1-\bar{q})r - \bar{q}(1-bx) = 0$$

But $\bar{q} - q_1$ is strictly positive and bounded away from zero. Therefore, the term $[(1-q_1)r - q_1(1-bx)]$ is strictly positive and bounded away from zero. Therefore, the positive-profit condition holds. We have thus found a contract (r', x') that breaks the supposed pooling equilibrium. ■

The intuition behind this result is that a borrower's subjective rate of substitution between interest and collateral is determined by his credit risk. The new contract changes the interest-collateral mix in a way that appeals only to low-risk borrowers, and thus drastically reduces the effective credit risk for the lender, so as to make it profitable. Since pooling equilibrium is impossible, this means in particular that it is impossible to sustain an efficient outcome in competitive equilibrium, because efficiency requires all borrower types to take a loan without collateral, which effectively means that they all must choose the same loan contract $(r, 0)$. It follows that if a competitive equilibrium exists, it must be inefficient.

Let us now turn to separating competitive equilibria. Since there are only two types, we can assume that C consists of two contracts: (r_1, x_1) denotes the contract that type 1 selects, and (r_2, x_2) denotes the contract that type 2 selects. Our task now is to characterize these contracts, assuming that a competitive equilibrium equilibrium exists.

Claim 3 $(r_2, x_2) = (\frac{q_2}{1-q_2}, 0)$. *That is, the contract aimed at type* 2 *mimics the complete-information benchmark.*

Proof. Assume the contrary - that is, $x_2 > 0$. In the complete-information case, we saw that such a contract is inconsistent with equilibrium, because a lender can enter the market and introduce a new contract (r_2', x_2') with r_2' slightly higher than r_2 and x_2' slightly lower than x_2, such that a borrower of type 2 would prefer (r_2', x_2') to (r_2, x_2), and the new contract would generate a positive profit when this borrower type chooses it. That is:

$$\begin{aligned} (1-q_2)r_2' + q_2 x_2' &> (1-q_2)r_2 + q_2 x_2 \\ (1-q_2)r_2' - q_2(1-bx_2') &> 0 \end{aligned}$$

In the incomplete-information case, a firm that offers such a new contract cannot prevent the other borrower type to choose this contract, such that the effective credit risk that characterizes the pool of borrowers who select the new contract will be $\bar{q}$. However, since $\bar{q} < q_2$, it follows that if this event transpires, it would only raise the lender's profit from the contract. Therefore, the new contract breaks the supposed equilibrium contract. ■

Thus, the transaction with high-risk types is efficient because it does not make use of collateral. Note that the expected utility that type 2 earns from this contract is

$$1 - (1 - q_2)r_2 - q_2 x_2 = 1 - (1 - q_2)\frac{q_2}{1 - q_2} = 1 - q_2$$

In other words, the expected payment associated with the contract is actuarially fair.

Let us now turn to the equilibrium contract selected by low-risk borrowers. Since $x_2 = 0$, and by the requirement that the equilibrium is separating, it must be the case that $x_1 > 0$. That is, the loan contract aimed at type 1 makes use of collateral.

Claim 4 *The equilibrium contract* (r_1, x_1) *selected by type* 1 *is the (unique) solution to the following pair of equations:*$1 - (1 - q_2)r_2 - q_2 x_2$

$$\begin{aligned} (1 - q_1)r_1 - q_1(1 - bx_1) &= 0 \\ (1 - q_2)r_1 + q_2 x_1 &= q_2 \end{aligned}$$

Proof. The first equation means that the contract generates zero profits when only borrowers of type 1 select it. This requirement follows immediately from the definition of a separating equilibrium. The second equation means that for type 2, the contract generates an expected payment of q_2. That is, type 2 is indifferent between (r_1, x_1) and (r_2, x_2). To see why the second equation must hold, suppose first that $(1-q_2)r_1+q_2 x_1 < q_2$. In this case, type 2 strictly prefers (r_1, x_1) to (r_2, x_2). This is inconsistent with the definition of (r_2, x_2) as the contract that type 2 selects in equilibrium. Alternatively, suppose that $(1 - q_2)r_1 + q_2 x_1 > q_2$. We already noted that $x_1 > 0$. In this case, a lender can enter with a new contract $(r_1', x_1') = (r_1 + \delta, x_1 - \varepsilon)$, such that - just as in the complete-information case - type 1 would strictly prefer the new contract to (r_1, x_1), and the new contract would generate positive profits from borrowers of type 1. If we set ε and δ to be small, the deviation from (r_1, x_1) is small enough that type 2 will continue to prefer his designated contract to the new contract, i.e. $(1 - q_2)r_1' + q_2 x_1' > q_2$. Thus, we found a new contract that enters the market and earns positive profits, and this breaks the equilibrium. It follows that in order for a separating equilibrium to exist, type 2 must be indifferent between the two contracts. ■

The intuition behind this result is that the incentives facing the high-risk type constrain the extent to which market competition can push in the direction of an efficient outcome. A new contract that caters to one risk group can attract other risk types and thereby change the effective credit risk of the pool of borrowers who select the new contract. This consideration does not affect the transaction with the high-risk type, because the "danger" that low-risk types will jump on the bandwagon and select a new contract aimed at the high-risk type is not a danger but a blessing that reduces the effective credit risk of the contract. Therefore, equilibrium transactions with the high-risk types will be efficient and involve no collateral.

On the other hand, transactions with the low-risk type are constrained by this consideration, because the possibility that a new contract that lowers collateral requirements and raises interest will attract the high-risk type is a potentially real danger for lenders, as it raises the effective credit risk of the new contract to such a degree that the new contract will make negative profits. Therefore, the equilibrium contract of the low-risk type is determined by the incentive constraint that high-risk borrowers will not pretend to be low-risk borrowers by selecting the contract aimed at the latter, in addition to the zero-profit condition. The incentive constraint requires the contract aimed at low-risk types to make use of collateral. In other words, collateral is a device for screening the low-risk borrowers in competitive equilibrium. This screening device entails inefficiency.

Comment: Equilibrium non-existence

Our analysis showed that if a competitive equilibrium exists, it must be separating, such that high-risk types are indifferent between the two contracts. However, this does not guarantee that this would indeed be an equilibrium. The reason is that when a lender offers a new contract $(r'_1, x'_1) = (r_1+\delta, x_1-\varepsilon)$, it is possible that the contract will generate positive profits even if type 2 abandons (r_2, x_2) and selects the new contract. In other words, it is possible that

$$(1-\bar{q})r'_1 - \bar{q}(1-bx'_1) > 0$$

and if this is the case, the new contract breaks the equilibrium. Such a scenario is likely when b is relatively small while q_1 is relatively close to q_2. On the other hand, if q_1 is much lower than q_2 and b is relatively close to one, such a scenario is unlikely and equilibrium will exist.

Comment: The composition of the population of borrowers

We assumed that the fraction of each type in the population of borrowers is 50%. However, note that for our characterization of equilibria, this assumption wasn't necessary. A pooling equilibrium fails to exist, regardless of the composition. And if a separating equilibrium exists, it is characterized by the parameters q_1, q_2, b, but the assumption we made regarding the fraction of high-risk types in the population of borrowers does not affect the equilibrium contracts.

The composition of the population of borrowers is, however, crucial for the issue of equilibrium existence. If the fraction of high-risk types is sufficiently low, it is easier to break a separating equilibrium because the possibility that a new contract aimed at low-risk types will also be chosen by high-risk type is less of a problem for a lender who offers such a contract. On the other hand, if the fraction of high-risk borrowers is sufficiently high, equilibrium existence is assured.

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