

# Consideration Sets and Competitive Marketing: Corrigendum\*

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## 1 Introduction

Eliaz and Spiegler (2011) (henceforth, ES) proposed a model of competitive marketing when consumers have limited propensity to consider all feasible market alternatives. A key result in the paper (Proposition 2) stated that there always exists a symmetric equilibrium in which firms earn the max-min profit. This statement turns out to be incorrect, and in this corrigendum we provide a necessary condition on the "consideration function" for the existence of an equilibrium with max-min payoffs for any "admissible" cost structure. Interestingly, this condition is based on the well-known mathematical concept of a "Helly family". We illustrate that the necessary condition is not sufficient, and also that the sufficient condition provided in Proposition 4 of ES is not necessary.

In this corrigendum we also address another, more minor mistake in ES. When costs are sufficiently small, firms earn max-min payoffs in any symmetric equilibrium, for essentially any consideration function. Proposition 6 in ES provided a bound on costs, below which firms earn max-min payoffs. The proof that appeared in ES contained a mistake, and here we restate the result with a slightly modified bound.

We begin by recalling the model of ES, using slightly different notation that would be useful for our current objective. Let  $X$  be a finite set of products, and let  $D$  be a finite set of "marketing devices". Two firms facing a

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single consumer play the following simultaneous-move game with complete information. A pure strategy for a firm is a pair  $(x, M)$ , where  $x \in X$  and  $M \subseteq D$ . Let  $c_x > 0$  and  $c_m > 0$  denote the fixed costs of  $x$  and  $m$ , respectively, and let  $c(x, M) \equiv c_x + \sum_{m \in M} c_m$  be the fixed cost of  $(x, M)$ . Faced with the strategy profile  $(x^i, M^i)_{i=1,2}$ , the consumer chooses according to a procedure based on two primitives: a strict preference relation  $\succ$  on  $X$  and a *consideration function*  $f : D \rightarrow 2^{X \setminus \{x^*\}}$ , where  $x^*$  is the  $\succ$ -maximal element in  $X$ , and  $f(m)$  is interpreted as the set of products from which  $m$  attracts attention. The consumer initially draws a firm  $i$  at random. He switches to firm  $j$  (and subsequently consumes  $x^j$ ) if and only if  $(x^j, M^j)$  “beats”  $(x^i, M^i)$ , which occurs whenever  $x^i \in \cup_{m \in M^j} f(m)$  and  $x^j \succ x^i$ . Each firm tries to maximize its probability of being chosen minus the fixed cost of its strategy. We focus on symmetric Nash equilibria. Given an equilibrium  $\sigma$  let  $Supp(\sigma)$  denote its support. Let  $\beta_\sigma(x)$  denote the probability that  $x$  is played in  $\sigma$ , namely  $\beta_\sigma(x) = \sum_M \sigma(x, M)$ .

The following conditions are imposed on the primitives. First,  $c_x \geq c_y$  whenever  $x \succ y$ , with a strict inequality when  $x = x^*$ . Second,  $c(x^*, D) < \frac{1}{2}$ . Third,  $\cup_{m \in D} f(m) = X$ . These conditions imply that the max-min payoff in the game is  $\frac{1}{2} - c_{x^*}$ . This naturally raises the question of whether firms are able to earn payoffs above this level in symmetric Nash equilibrium. This is an important question, for two reasons. First, the max-min payoff is also the equilibrium payoff that firms earn if consumers are fully rational in the sense that they always consider the entire feasible set of market alternatives, independently of the firms’ marketing strategies. Second, max-min equilibrium payoffs imply an interesting corollary regarding consumers’ conversion rates on the equilibrium path, a property referred to as the Effective Marketing Property.

Proposition 2 in ES stated that for a tuple  $(c, f, \succ)$  with the above properties, there exists a symmetric mixed-strategy Nash equilibrium in which firms earn max-min payoffs. However, the constructive proof of this claim failed to take into account certain deviations to pure strategies outside the support of the putative equilibrium strategy.

## 2 Necessary condition

For every non-empty  $Y \subseteq X$ , denote  $f_Y(m) = f(m) \cap Y$ . A collection  $\{X^k\}_{k=1, \dots, K}$  of subsets of  $X$  is a Helly family, if whenever  $X^k \cap X^{k'} \neq \emptyset$  holds for any  $k, k' \in \{1, \dots, K\}$ , then  $\cap_{k=1, \dots, K} X^k \neq \emptyset$ .<sup>1</sup> We say that a

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<sup>1</sup>See [http://en.wikipedia.org/wiki/Helly\\_family](http://en.wikipedia.org/wiki/Helly_family).

consideration function  $f$  satisfies the **Helly** property if for every non-empty subset of marketing devices  $\{m^1, \dots, m^K\} \subseteq D$  and every non-empty subset of products  $Y \subseteq X \setminus \{x^*\}$ , the collection of subsets  $\{f_Y(m^k)\}_{k=1, \dots, K}$  is a Helly family.

**Proposition 1** *If  $f$  violates the Helly property, then the rational-consumer payoff  $\frac{1}{2} - c_{x^*}$  is unsustainable in symmetric Nash equilibrium for generic permissible cost structures.*

**Proof.** We construct a proof by contradiction that proceeds in three steps. Let  $\sigma$  be a symmetric Nash equilibrium.

**Step 1:** If  $f$  violates the Helly property, then there exists a set of three marketing devices  $M^3 = \{m^1, m^2, m^3\} \subseteq D$  and a set of three inferior products  $X^3 = \{x^1, x^2, x^3\} \subseteq X \setminus \{x^*\}$  such that  $\{f_{X^3}(m^k)\}_{k=1,2,3}$  is *not* a Helly family.

**Proof:** If  $f$  violates the Helly property, then there is a set of marketing devices  $M' \subseteq D$  and a set of products  $Y' \subseteq X$  such that  $f_{Y'}(m) \cap f_{Y'}(m') \neq \emptyset$  holds for every  $m, m' \in M'$ , but  $\cap_{m \in M'} f_{Y'}(m) = \emptyset$ . Among these pairs  $(M', Y')$ , select a pair  $(M, Y)$  with a minimal  $M$  — that is, there exists no  $(M', Y')$  as defined above such that  $M' \subset M$ . Therefore,  $\cap_{m' \in M \setminus \{m\}} f_Y(m') \neq \emptyset$  for every  $m \in M$ . Clearly,  $|M|, |Y| \geq 3$ . Impose an arbitrary enumeration on  $M$ , such that  $M = \{m^1, \dots, m^K\}$ ,  $K \geq 3$ . By the minimality of  $M$ , for every  $m^k \in M$  there is  $x^k \in Y$  such that  $x^k \notin f_Y(m^k)$  and  $x^k \in \cap_{m \in M \setminus \{m^k\}} f_Y(m)$ . Define  $M^3 = \{m^1, m^2, m^3\} \subseteq M$  and  $X^3 = \{x^1, x^2, x^3\}$ . By definition,  $f_{X^3}(m^1) = \{x^2, x^3\}$ ,  $f_{X^3}(m^2) = \{x^1, x^3\}$  and  $f_{X^3}(m^3) = \{x^1, x^2\}$ , hence the collection  $\{f_{X^3}(m^k)\}_{k=1,2,3}$  is not a Helly family.  $\square$

Let  $\alpha_\sigma(m)$  denote the probability that a marketing device  $m$  is played in  $\sigma$ , i.e.,

$$\alpha_\sigma(m) \equiv \sum_{(x, M) \in \text{Supp}(\sigma) | m \in M} \sigma(x, M)$$

**Step 2:** For any  $\epsilon > 0$ , there exists a generic permissible cost structure such that for any symmetric Nash equilibrium strategy  $\sigma$  that induces max-min payoffs,  $\alpha_\sigma(m) \leq \epsilon$  for every  $m \notin M^3$ .

**Proof:** Denote  $X_3^c = X \setminus (X^3 \cup \{x^*\})$ , and denote  $M_3^c = D \setminus M^3$ . Let  $c_{x^k} = \bar{c} + \epsilon_k$  for every  $x^k \in X^3$ , where  $\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$ ,  $c_{x^*} > \bar{c} + \frac{1}{3} + \epsilon |D|$  and let  $c_x \geq c_{x^*} - \epsilon/2$  for every  $x \in X_3^c$ , where  $\epsilon > 0$ . Set  $c_{m^k} = \bar{c} + \epsilon_k$  for

every  $m^k \in M^3$ , and let  $c_m > 3\tilde{c}$  for every  $m \in M_3^c$ . Clearly,  $\epsilon, \epsilon_1, \epsilon_2, \epsilon_3$  must all be sufficiently small in order to ensure the cost structure is permissible, namely that  $c(x^*, D) < \frac{1}{2}$  and that  $c_{x^*} > c_x$  for all  $x \neq x^*$ .

Assume there exists a symmetric Nash equilibrium  $\sigma$  in which firms earn max-min payoffs. Assume that  $\alpha_\sigma(m) > \epsilon$  for some  $m \in D$ . Suppose there exists  $(x', M') \in \text{Supp}(\sigma)$  such that  $x' \in f_{X_3^c}(m)$ . By the Effective Marketing Property (Proposition 5 in ES),  $(x', M')$  is beaten by any  $(x'', M'') \in \text{Supp}(\sigma)$  with  $m \in M''$ . Hence, by playing  $(x^*, M')$  instead of  $(x', M')$  a player would increase his market share by at least  $\frac{1}{2}\alpha_\sigma(m)$ , while increasing his cost by less than  $\frac{1}{2}\epsilon < \alpha_\sigma(m)$ . It follows that for any marketing device  $m$  with  $\alpha_\sigma(m) > \epsilon$ , the only products in  $f(m)$  that are played with positive probability in  $\sigma$  are those in  $X^3$ . Suppose  $\alpha_\sigma(m) > \epsilon$  for some  $m \in M_3^c$  and consider some  $(\hat{x}, \hat{M}) \in \text{Supp}(\sigma)$  for which  $m \in \hat{M}$ . Then, by switching from  $(\hat{x}, \hat{M})$  to  $(\hat{x}, (\hat{M} \setminus \{m\}) \cup M^3)$ , a firm reduces its cost without lowering its market share.  $\square$

**Step 3:** Firms earn more than  $\frac{1}{2} - c_{x^*}$  in any symmetric Nash equilibrium for a generic permissible cost structure.

**Proof:** Consider the cost structure assumed at the beginning of the proof of Step 2, and assume that there exists a symmetric Nash equilibrium strategy  $\sigma$  that induces max-min payoffs. Then,  $(x^*, \emptyset)$  is a best response to  $\sigma$ . Thus, for any  $x^k \in X^3$ , the strategy  $(x^k, \emptyset)$  cannot achieve a higher payoff against  $\sigma$  than  $(x^*, \emptyset)$ . This means that if a player switched from playing  $(x^*, \emptyset)$  against  $\sigma$  to playing  $(x^k, \emptyset)$ , the expected loss in market share would be weakly greater than the savings in costs. Therefore,

$$\frac{1}{2} \sum_{m \in M^3 \setminus \{m^k\}} \alpha_\sigma(m) + \frac{1}{2} \sum_{m \in M_3^c} \epsilon \geq c_{x^*} - \bar{c} - \epsilon_k$$

To see why this inequality holds, recall that by Step 2, only marketing devices in  $M^3$  are chosen with a probability strictly greater than  $\epsilon$  in any equilibrium with max-min payoffs. Therefore, the left hand side of the above inequality is an upper bound for the probability that a consumer's attention is attracted by the opponent's marketing strategy, and hence, the left side is an upper bound for the loss in market share. Summing up these inequalities over all  $x^k \in X^3$  yields

$$\sum_{m^k \in M^3} \alpha_\sigma(m^k) + \frac{3}{2} \sum_{m \in M_3^c} \epsilon \geq 3(c_{x^*} - \bar{c})$$

Since  $c_{x^*} - \bar{c} > \frac{1}{3} + \epsilon|D|$ , it follows that  $\sum_{m^k \in M^3} \alpha_\sigma(m^k) > 1 + \frac{3}{2}\epsilon|D|$ . This means that there exists some  $M$  with  $\sum_{x \in X \mid M \subseteq M'} \sigma(x, M') > \epsilon$  and

$|M \cap M^3| \geq 2$  - otherwise, the sum of all elements in  $Supp(\sigma)$  would be strictly greater than one, a contradiction. Assume w.l.o.g. that  $\{m^1, m^2\} \subseteq M$ . Recall that in the proof of Step 2, we showed that for any marketing device  $m$  with  $\alpha_\sigma(m) > \epsilon$ , the only products in  $f(m)$  that are played with positive probability in  $\sigma$  are those in  $X^3$ . Since  $f_{X^3}(m^1) \cup f_{X^3}(m^2) = X^3$ , it follows that  $M = \{m^1, m^2\}$  - otherwise, a firm could deviate from  $(x, M)$  to  $(x, \{m^1, m^2\})$  and lower its cost without lowering its market share.

As firms earn by assumption max-min payoffs, and  $x^3 \in f(m^1)$  and  $x^3 \in f(m^2)$  holds, Proposition 3 in ES implies that  $x^3$  is not played at all in  $\sigma$ . But this means that a firm could deviate from  $(x, \{m^1, m^2\})$  to  $(x, \{m^3\})$  and lower its cost without lowering its market share. Hence  $(x, \{m^1, m^2\})$  is not a best-reply to  $\sigma$ , a contradiction. ■

Let us now illustrate that the necessary condition for max-min equilibrium payoffs is not sufficient, and that the sufficient condition provided by Proposition 4 in ES, namely that the consideration function is partitional, is not necessary.

*Necessary condition is not sufficient*

Let  $X = \{x^1, \dots, x^K\} \cup \{x^*\}$  and  $D = \{m^1, \dots, m^K\}$ . Define the consideration function  $f_K$  as follows:

$$f_K(m^k) = \{x^{k \bmod K}, x^{(k+1) \bmod K}\} \quad (1)$$

Note that  $f_3$  violates the Helly property. Hence, symmetric equilibrium profits exceed the max-min for generic permissible cost structures.

This example also illustrates the non-monotonicity of equilibrium profits with respect to consumer attention. Fix  $X = \{x^1, x^2, x^3\} \cup \{x^*\}$  and  $D = \{m^1, m^2, m^3\}$ . As pointed out in ES, one could imagine a scale that measures consumers' resistance to considering new alternatives. At one end of the scale is the rational consideration function  $f_R(m) = \{x^1, x^2, x^3\}$  for all  $m \in D$ , and at the other end of the scale there is the partitional consideration function  $f_P$  defined by  $f_P(m^k) = \{x^k\}$ . In both cases, symmetric equilibrium profits are equal to the max-min. The function  $f_3$  is "in between" these two extremes (in terms of the consumer's propensity to consider new alternatives), and yet it induces equilibrium payoffs above the max-min for generic permissible cost structures.

Now consider  $f_5$ . This consideration function vacuously satisfies the Helly property, and yet it can be shown that symmetric equilibrium payoffs must exceed the max-min for generic permissible cost structures, using a construction similar to that in the proof of Proposition 1.

*Sufficient condition is not necessary*

Proposition 4 in ES shows that if  $\{f(m)\}_{m \in M}$  is a partition of  $X \setminus \{x^*\}$ , then firms earn max-min payoffs in every symmetric Nash equilibrium. Consider the following specification of the model. Let  $X \setminus \{x^*\} = \{1, 2, 3, 4\}$ ,  $D = \{m_1, m_2, m_3\}$ ,  $f(m_k) = \{k, 4\}$  for every  $k = 1, 2, 3$ . Assume  $x \succ 4$  for every  $x \neq 4$ . The consideration function is clearly non-partitional, but it satisfies the Helly property. Let us now show that firms earn the max-min in every symmetric Nash equilibrium, for any permissible cost structure.

Fix a symmetric equilibrium strategy  $\sigma$ . By Lemma 1 in ES,  $\beta_\sigma(x^*) > 0$ . If  $(x^*, \emptyset) \in \text{Supp}(\sigma)$ , we are done. Suppose  $(x^*, \emptyset) \notin \text{Supp}(\sigma)$ . If  $\beta_\sigma(4) > 0$ , then 4 is beaten by every  $(x^*, M) \in \text{Supp}(\sigma)$ . Since 4 is the  $\succ$ -minimal product,  $(4, \emptyset) \in \text{Supp}(\sigma)$  and this alternative does not beat any element in  $\text{Supp}(\sigma)$ . It follows that a firm deviates from  $(4, \emptyset)$  to  $(x^*, D)$ , it increases its market share by at least  $\frac{1}{2}\beta_\sigma(x^*) + \frac{1}{2}(1 - \beta_\sigma(x^*)) = \frac{1}{2} > c(x^*, D) - c(4, \emptyset)$ , hence the deviation is profitable. Therefore,  $\beta_\sigma(4) = 0$ . But this means that the equilibrium must be the same as if 4 were eliminated from  $X$ , in which case  $f$  would be partitional. By Proposition 4 in ES, symmetric equilibrium payoffs in this case are equal to the max-min.

### 3 Max-min payoffs for sufficiently small costs

Proposition 6 of ES stated that if  $c(x^*, D) < 1/(2^{|D|} + 2)$ , then firms would earn the max-min payoffs in any symmetric equilibrium. The proof pointed out that if firms earn above the max-min payoff at some equilibrium  $\sigma$ , then any  $(x^*, M) \in \text{Supp}(\sigma)$  must beat some  $(x', M') \in \text{Supp}(\sigma)$ . Since  $(x', M')$  is a best-reply to  $\sigma$ , it cannot be profitable to deviate from  $(x', M')$  to  $(x^*, M)$ . In ES, we translated this observation to the following inequality:

$$\frac{1}{2}c(x^*, M) + \frac{1}{2} \sum_{x \prec x^*} \beta_\sigma(x) \leq c(x^*, M) - c(x', M')$$

This inequality, however, is incorrect because it ignores the possibility that some strategies are beaten by *both*  $(x^*, M)$  and  $(x', M')$ .

The following result is a restatement of Proposition 6 in ES, with a slightly lower upper bound on costs. When costs are below this bound, firms earn the max-min payoff in any symmetric equilibrium.

**Proposition 2** *Let  $m^*$  be the most costly marketing device. If*

$$(2^{|D|} - 1) \cdot c_{x^*} + (|X| - 1) \cdot c_{m^*} < \frac{1}{2} \quad (2)$$

then firms earn the rational-consumer payoff in any symmetric Nash equilibrium.

**Proof.** Assume (2) holds. Let  $\sigma$  be a symmetric Nash equilibrium in which firms earn above the max-min payoff. By Step 1 of Proposition ??, for every  $(x, M) \in \text{Supp}(\sigma)$  with  $x \neq x^*$ , there exists  $(x^*, M') \in \text{Supp}(\sigma)$  such that  $(x^*, M')$  does not beat  $(x, M)$ . By assumption (P2) in ES, there exists some  $m(x) \in D$  such that  $x \in f(m)$  and hence,  $(x^*, M' \cup \{m(x)\})$  would beat  $(x, M)$ . Since  $(x^*, M') \in \text{Supp}(\sigma)$ , it follows that

$$\frac{1}{2}\beta_\sigma(x) \leq c_{m(x)}$$

since otherwise, it would be strictly profitable to deviate from  $(x^*, M')$  to  $(x^*, M' \cup \{m(x)\})$ . Summing these inequalities over all  $x \prec x^*$  yields:

$$\frac{1}{2} \sum_{x \prec x^*} \beta_\sigma(x) \leq \sum_{x \prec x^*} c_{m(x)} \leq (|X| - 1)c_{m^*} \quad (3)$$

Let  $A(x^*, M)$  denote the set of strategies  $(x', M') \in \text{Supp}(\sigma)$  that are beaten by  $(x^*, M)$ . Let  $a(x', M') \in A(x^*, M)$ . Because firms earn above the max-min payoffs,  $A(x^*, M) \neq \emptyset$  for all  $(x^*, M) \in \text{Supp}(\sigma)$ . In addition, for each  $(x^*, M) \in \text{Supp}(\sigma)$ , it is not profitable to deviate from any  $a(x^*, M)$  to  $(x^*, M)$ , hence

$$\frac{1}{2}\sigma(x^*, M) \leq c_{x^*} - c_{a(x^*, M)}$$

Since by assumption,  $(x^*, \emptyset) \notin \text{Supp}(\sigma)$  (firms earn above max-min payoffs), summing over all strategies  $(x^*, M) \in \text{Supp}(\sigma)$  we obtain

$$\frac{1}{2}\beta_\sigma(x^*) < (2^{|D|} - 1) \cdot c_{x^*} \quad (4)$$

Summing (3) and (4) yields:

$$\frac{1}{2} < (2^{|D|} - 1) \cdot c_{x^*} + (|X| - 1) \cdot c_{m^*}$$

a contradiction. ■

## References

- [1] Eliaz, K. and R. Spiegler (2011), "Consideration Sets and Competitive Marketing," *Review of Economic Studies* 78, 235-262.