Buridanic Competition*

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Abstract

We analyze a model of two-attribute competition for a decision maker who follows a non-compensatory choice procedure that only responds to ordinal rankings along the two dimensions. The decision maker has an outside option that functions as a default alternative. In the absence of a dominant alternative, the decision maker may stick to the default even if it is dominated - capturing the phenomenon of choice procrastination in the presence of difficult choices. We show that the prevalence of difficult-choice situations in equilibrium is related to the magnitude of the choice procrastination effect. In general, features of the choice procedure that are typically viewed as biases tend to "protect" the decision maker, in the sense that they encourage competitors to offer higher-value alternatives in equilibrium. We discuss the potential implications of this analysis for recent discussions of "default architecture".

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1 Introduction

One of the biggest distinctions between economists' and psychologists' view of decision processes is the way they regard trade-offs. The standard economic approach assumes that the decision maker (DM henceforth) has well-defined preferences, and in the vast majority of applications these preferences are continuous and locally non-satiable. The economic DM is a "trade-off machine" who effortlessly weighs multiple considerations – Kreps (1988) fondly calls him Totrep ("Trade-Off Talking Rational Economic Person").

A viewpoint more typical of psychologists (Tversky (1972), Payne et al. (1993), Luce et al. (1999), Anderson (2003)) is that the DM generally tries to *avoid trade-offs* because of the cognitive and emotional difficulty of performing them. This motive is stronger when the DM needs to *justify* his choices to other people, because decision weights are hard to account for. A trade-off avoiding DM will employ so-called "non-compensatory" choice procedures that rely purely on the *ordinal* rankings over alternatives along each dimension. And if the DM has a default option that enables him to "decide not to decide", he may exercise this option and thus "save the mental cost" of actively resolving trade-offs.

This paper explores the possible implications of trade-off avoidance for *competitive interaction* in market or organizational contexts. Does it lead to non-competitive equilibrium outcomes? If so, how large is the departure of from the rational-DM competitive benchmark? How often is the DM called to perform trade-offs in equilibrium? What is the role of the default specification for these questions?

To address these questions, we study a simple model in which two agents compete in two-dimensional alternatives for a DM. We refer to these agents as "competitors". The alternative offered by competitor i is a pair $q_i = (q_i^1, q_i^2) \in \mathbb{R}_+^2$, where q_i^k measures the alternative's "true value" along dimension k. For instance, q_i^1 can represent the alternative's quality whereas q_i^2 is a decreasing function of its price. In other applications, both dimensions represent aspects of the alternative's quality - e.g., a car can be characterized by its safety and energy efficiency. A dimension may also correspond to a state of Nature, such that q_i^k describes an investment project's performance in state k.

The DM's choice set consists of q_1 and q_2 - also referred to as the "*market alternatives*" - as well as an exogenous outside option $q_0 = (q_0^1, q_0^2)$. For most of the paper, the outside option is also the DM's *default* alternative - i.e., the one he ends up with if he fails to make an explicit/active decision. Competitor i's payoff is $2 - q_i^1 - q_i^2$ multiplied by the probability it is chosen by the DM. (Note that q_i^k thus quantifies the alternative's value along dimension k in terms of the competitor's cost of providing this value.) We refer to the probability that the DM switches away from the outside option and chooses one of the competitors' alternatives as his "*market participation rate*".

A conventionally rational DM would be endowed with a continuous, strictly increasing function $u(q^1, q^2)$, and he would choose an alternative that maximizes u. In this case, our model would collapse into conventional, Bertrand-like competition: in Nash equilibrium, competitors would offer alternatives that maximize u subject to the zero-profit constraint $q^1 + q^2 = 2$ (as long as the outside option has a sufficiently low u value).

In contrast, the DM in our model follows a *non-compensatory* probabilistic choice function, which is based solely (in a monotonically increasing manner) on *ordinal rankings* along the two dimensions. When one alternative dominates all others, we assume the DM chooses it with probability one. In the absence of a dominant alternative, we say that the DM faces a "*difficult choice*". The DM may resist the pressure to perform trade-offs by pretending that it does not exist and neglecting one of the attributes, thus basing his choice entirely on the remaining one. Alternatively, he may procrastinate ("decide not to decide") and thus end up with his default option - even if it is dominated by the other available alternatives.

The latter scenario is reminiscent of the proverbial *Buridan's Ass*: the

DM finds it hard to trade-off the relative strengths and weaknesses of the undominated alternatives; his hesitation causes him to procrastinate, and with some probability this procrastination leaves him stuck with the default option in the relevant time frame. "Buridanic" situations of this nature have received considerable attention by researchers who studied empirically choice procrastination in the presence of hard choice problems, in both experimental and "field" settings, notably retirement savings (Iyengar et al. (2004), Madrian and Shea (2001) and Beshears et al. (2012)). At any rate, we assume that the DM never chooses a *dominated non-default* alternative; in this sense, our model captures *default bias*.

In its basic features, our model fits market environments in which firms compete in multi-attribute products. For example, think about weighing a car's safety against its energy efficiency. Even if the consumer has access to precise data about each attribute, it may be hard for him to find the right scale for comparison. The difficulty is not only cognitive but also emotional, because the consumer ultimately needs to trade off the risk of injury or death against lower fuel costs. Similarly, when consumers choose between insurance policies that provide different levels of coverage in different contingencies, they have to perform complex actuarial calculations and imagine unpleasant future events.

A different interpretation holds in organizational settings. For instance, the DM can be a company official who considers several candidates for a construction job with several dimensions (total cost, speed of delivery, quality of materials, etc.). The official needs to justify the selection to his superiors. When the selection is not obvious and requires the exercise of judgment, the official is more vulnerable to criticisms. It may be easier for him to justify his choice to his superiors if he leaves out relevant dimensions, or if he opts for the same constructor that the company has employed before.

Competitors in this model face two conflicting strategic considerations. On one hand, there is a "competitive" motive to offer a dominant alternative in order to win the DM over. On the other hand, domination requires the competitor to offer high value along both dimensions, and this is a costly strategy; a cheaper course of action is to offer a "lopsided" location in $\mathbb{R}^2_+$ - e.g., low q^1 and high q^2 - such that the chances of being dominated by the opponent are low. The latter is an "obfuscatory" or "anti-competitive" motive. It follows that the question of the competitiveness of the game's outcome is related to the question of how frequently the DM faces market alternatives that dominate one another. In Section 3, we begin our analysis of symmetric Nash equilibria in the game by exploring the latter question.

Suppose that the choice function has the property that the market participation rate increases when one market alternative dominates another. This condition tends to hold in "Buridanic" situations, where domination removes the DM's hesitation and thus overcomes his tendency to adhere to the default/outside option. We show that in this case, domination between market alternatives must occur with positive probability in equilibrium. We also prove a partial converse result. If the choice function satisfies the property that domination between market alternatives never raises the market participation rate, then domination can never occur in equilibrium between two alternatives that belong to same quadrant relative to the outside option. In particular, when the outside option is $(0, 0)$, the DM always faces difficult choices in equilibrium. The collection of results in Section 3 thus relates the possibility of "easy choices" in equilibrium to a simple property of the DM's choice function.

In Section 4, we use the general results of Section 3 to get more detailed characterizations of symmetric Nash equilibria for various natural specifications of the DM's outside option and his choice procedure. We also use the examples to illustrate the various interpretations of the value dimensions in our model. A general theme that emerges from these exercises is that features of the DM's choice procedure that are typically viewed as "biases" tend to "protect" him in competitive environments - in the sense that in equilibrium, they lead competitors to offer alternatives with higher average value $\frac{1}{2}(q^1 + q^2)$. For example, when the DM avoids trade-offs by focusing on a random dimension, greater asymmetry of this distribution over dimensions corresponds to a "salience bias". Yet it also leads to a more competitive equilibrium outcome. Likewise, when the DM is more likely to avoid trade-offs by adhering to his default/outside option, the range of possible equilibrium outcomes (in a class of equilibria we were able to identify) moves toward the competitive benchmark.

Our modeling framework also enables us to study the equilibrium implications of the design of default rules. Default architecture is one of the most important policy ideas that have come out of behavioral economics (see Thaler and Sunstein (2008)). However, to our knowledge this is the first paper that analyzes the equilibrium effects of default architecture in the context of an explicit behavioral model that generates default bias.[1] The default rule we assume for most of the paper is what the literature calls "*opt in*" - i.e., the default is the outside option. In Section 5, we compare it to a rule known as "opt out" - i.e., the DM is initially assigned to a market alternative. We revisit the examples of Section 4 and show that unsurprisingly, the switch from "opt in" to "opt out" raises the market participation rate. However, it also leads to an anti-competitive equilibrium effect, by weakening the pressure to offer dominant alternatives. In some cases, the switch may have a negative net effect on the DM's welfare. We discuss the possible implications of these results for contemporary discussions of default architecture. In particular, we suggest that in settings like retirement saving, the success of the "soft paternalism" of default architecture owes to a complementary "hard paternalism" regulatory regime that effectively shuts down adverse equilibrium effects.

[1]Spiegler (2015) contains a similar exercise, in the context of a different modeling framework. However, that paper postdated an earlier version of the present paper.

2 The Model

Two competitors play a symmetric simultaneous-move, complete information game. Each competitor $i = 1, 2$ offers an alternative characterized by a pair $(q_i^1, q_i^2) \in \mathbb{R}_+^2$. For expositional simplicity, we refer to q_i^k as the "quality" of alternative i along dimension k. We say that q dominates r if $q > r$ (i.e., $q^k \geq r^k$ for both $k = 1, 2$, with at least one strict inequality). The competitors face a single DM whose choice set consists of the competitors' alternatives (also referred to as "market alternatives") and an exogenous outside option, denoted 0 and represented by the quality pair $(q_0^1, q_0^2) \in \mathbb{R}_+^2$. The outside option is also the DM's *default* option (we relax this assumption in Section 5). The DM chooses according to a probabilistic choice function s, such that $s_i(q_0, q_1, q_2)$ is the probability that the DM chooses alternative $i \in \{0, 1, 2\}$. Competitors are expected-profit maximizers. Competitor i's payoff given the strategy profile (q_1, q_2) is $[2 - (q_i^1 + q_i^2)] \cdot s_i(q_0, q_1, q_2)$.

We impose the following assumptions on s.

(i) *Ordinality.* The choice function is invariant to changes in (q_0, q_1, q_2) that leave the ordinal rankings along each dimension unchanged.

(ii) *Competitor symmetry.* The choice function is neutral to the competitors' labels. That is, $s_1(q_0, q, r) = s_2(q_0, r, q)$ for every $r, q \in \mathbb{R}_+^2$.

(iii) *Monotonicity.* For any $i = 0, 1, 2$, the probability that alternative i is chosen is weakly increasing in its ordinal ranking along any dimension. In particular, if $q_i > q_j$ for all $j \neq i$, then $s_i(q_0, q_1, q_2) = 1$. For some results, we will assume *strict monotonicity* - i.e., the probability that an undominated alternative is chosen is strictly increasing in its ordinal position along any dimension.

(iv) *Exclusion of dominated non-default alternatives.* If $q_i < q_j$ for some $i \in \{1, 2\}$ and $j \in \{0, 1, 2\}$, then alternative i is chosen with zero probability.

Ordinality is the key assumption of this paper: it reflects the DM's extreme reluctance to carry out trade-offs. Competitor symmetry ensures the game's symmetry. Monotonicity simply means that a higher ranking along any dimension raises (weakly or strictly) the probability of being selected; and when one alternative dominates all others, it is chosen with probability one. The final property means that a market alternative is never selected if it is dominated. Note that while the choice function s is neutral to the competitors' labels, it is not neutral to the distinction between the market alternatives and the outside/default option. This enables us to capture reference-point effects. In particular, property (iv) allows the DM to select the outside option even when it is dominated by one of the market alternatives.

The following specifications of s will appear in later sections.

Sampling a dimension. In the absence of a dominant alternative, the DM ignores one dimension at random and selects the best alternative along the remaining dimension (with arbitrary tie breaking). One interpretation is that since the DM does not know how to weigh the two dimensions, he simply neglects one of them. This choice procedure is behaviorally equivalent to the maximization of a random utility function ($u(q^1, q^2) = q^1$ with probability α and $u(q^1, q^2) = q^2$ with probability $1 - \alpha$).

Choice procrastination. In the absence of a dominant alternative, the DM chooses the outside/default option with probability λ, and follows the "sampling a dimension" rule with probability $1 - \lambda$. The interpretation is that the DM's hesitation over how to choose when there is no clear winner results in choice procrastination, such that with some probability the DM will fail to reach a decision in the relevant time frame. In that case, he will be stuck with the outside option by default.

Because the DM's choice function is generally not based on utility maximization, his choices do not reveal welfare in the traditional revealed-preference sense. Therefore, welfare analysis should be carried out with caution. As a convention, we will measure the DM's "true" welfare according to the average quality $\frac{1}{2}(q_i^1 + q_i^2)$ of the alternative i he ends up choosing, regardless of his default alternative. Although our procedural model can be linked to some "mental cost" of making active decisions in complex choice problems this cost is not explicit in our model. Moreover, it is far from clear how one could incorporate such a cost in a non-trivial manner, without fundamentally subverting our entire modeling approach. Therefore, we leave such mental costs outside the welfare analysis. (This methodological difficulty is familiar in the literature - see Spiegler (2011).)

3 Equilibrium Occurrence of Easy Choices

We now turn to analysis of symmetric Nash equilibria in the game described in Section 2. We take it for granted throughout that s satisfies properties $(i)-(iv)$. In order to avoid trivial equilibria, we also assume that there exist q , r such that $q^1+q^2 < 2$, $r^1+r^2 < 2$ and $s_0(q_0, q, r) \neq 1$. That is, the outside option and the DM's choice function leave scope for market participation.

The analysis is based on the following key distinction regarding s: *Does domination between market alternatives lower the probability that the DM chooses the outside option*? We begin by assuming that it does not. Specifically, assume that for every $q_0, q, r \in \mathbb{R}^2_+$ where $r > q$ it holds that $s_0(q_0, q, r) \geq s_0(q_0, (q^1, r^2), (r^1, q^2))$. Two examples in which this condition holds are when the DM is unable to choose the outside option, or when the DM's tendency to choose the default option is independent of the ordinal ranking of the market alternatives.

Our first result establishes that when s satisfies this property (as well as strict monotonicity), any symmetric Nash equilibrium has the feature that whenever the market alternatives have the same ordinal ranking w.r.t the outside option along both dimensions (i.e., they belong to the same quadrant relative to q_0), no alternative ever dominates another.

Proposition 1 *Assume s satisfies strict monotonicity, and $s_0(q_0, q, r) \geq s_0(q_0, (q^1, r^2), (r^1, q^2))$ whenever $r > q$. Then, in any symmetric Nash equi-*

librium, $q \not> q'$ for every two realizations q, q' of the equilibrium strategy that belong to same quadrant relative to q_0.

The key argument in the proof is that if there were two quality pairs q and r in the support of the equilibrium strategy that dominate one another, then deviating to (q^1, r^2) or (r^1, q^2) would be profitable (see Figure 2 in the Appendix). Consider the effect of deviating from q to r, (q^1, r^2) or (r^1, q^2) on a competitor's payoff. Depending on the opponent's strategy, all three deviations increase demand (i.e., the probability of being chosen) as well as the cost of provision. The model's assumptions - and particularly the assumption that domination does not increase participation - imply that the increase in demand when shifting from q to r is weakly lower than the sum of demand increases due to deviations from q to (q^1, r^2) and from q to (r^1, q^2). At the same time, the cost increase due to shifting from q to r is by definition equal to the sum of cost increases due to deviations from q to (q^1, r^2) and from q to (r^1, q^2). These observations imply the net first-order effect of shifting from q to r on a competitor's payoff is weakly lower than the sum of the net first-order effects of shifting from q to (q^1, r^2) and from q to (r^1, q^2). The second-order effects, however, ensure a strict inequality: incurring a large cost increase with high probability (when deviating from q to r) is less profitable than incurring two lower cost increases with lower probability (when deviating from q to (q^1, r^2) or (r^1, q^2)). If both q and r are in the support of the equilibrium strategy, then the total effect of shifting from q to r on profits is zero, which implies that a shift from q to (q^1, r^2) or (r^1, q^2) would be strictly profitable.

When $q_0 = (0, 0)$, market alternatives necessarily dominate the outside option, and therefore the quadrant qualification in Proposition 1 can be removed. This leads to the following characterization.

Proposition 2 *If $q_0 = (0, 0)$, then under the conditions of Proposition 1, for any symmetric Nash equilibrium, there exist $\bar{q}^1, \bar{q}^2 > 0$ such that the support* *of the equilibrium strategy is a continuous and strictly decreasing curve that connects the points* $(0, \bar{q}^2)$ *and* $(\bar{q}^1, 0)$*.*

What is the significance of this result? Note that one interpretation of our model is that making difficult choices involves a mental cost, which the DM could successfully avoid if he were allowed to choose by default. Proposition 2 means that whenever market alternatives dominate the outside option, spontaneous competitive forces "conspire" to maximize this mental cost.

We now turn to the case in which the DM's choice function satisfies the property that the market participation rate increases when one market alternative dominates another. We show that in this case, easy choices must occur with positive probability in any symmetric Nash equilibrium. (For this result, we do not need to impose strict monotonicity.)

Proposition 3 *Assume* $s_0(q_0, q, r) < s_0(q_0, (q^1, r^2), (r^1, q^2))$ *whenever* $q > r$*. Then, in any symmetric Nash equilibrium, market alternatives dominate one another with positive probability.*

The idea of the proof is the mirror image of the key argument behind the proof of Proposition 1. If the support of the equilibrium strategy does not contain alternatives that dominate one another, then we can find two quality pairs q and r in the support, such that deviating to (q^1, r^2) or (r^1, q^2) is profitable. However, observe that Proposition 3 is not an exact converse of Proposition 1, because the latter focuses on *strictly* monotone choice functions and examines the possibility of easy choices only among market alternatives that share the same ordinal ranking relative to the outside option. Later on we will present an example that demonstrate that the gap between the two results is not vacuous.

4 Applications

In this section we analyze symmetric Nash equilibria for two specifications of s that capture different underlying choice procedures. Throughout the section, we commit to a particular interpretation of the game and a particular specification of q_0. The DM is a procurement officer in a public organization. He needs to select a provider of some service having two quality dimensions. (A fixed budget is exclusively devoted to this service. Therefore, the service price is not a consideration for the officer, as long as it is within the budget.) The outside/default option is not to acquire the service; it is characterized by the quality pair $q_0 = (0, 0)$.

4.1 Sampling a Dimension

Suppose that the procurement officer chooses according to the "sampling a dimension" procedure described in Section 2: when one alternative dominates all others, he selects it; otherwise, he samples dimension k with probability α^k and selects the alternative with the highest quality along that dimension (with a symmetric tie-breaking rule). The choice function induced by this procedure satisfies the conditions for Proposition 2.

One interpretation for the officer's procedure is that he has to justify his actions to a superior who does not know the relevant considerations for choosing service providers but expects to hear a good reason for the officer's choice. The officer is not equipped with formal guidelines about how to trade-off the two quality dimensions, and therefore he finds it easier to justify his choices by neglecting one of the two quality dimensions. Over the long run, he must be consistent in this selection - he cannot neglect different attributes in different instances of this decision problem.

The parameter α^k captures the salience of dimension k - a more salient attribute is less likely to be neglected. W.l.o.g, let $\alpha^1 = \alpha \geq \frac{1}{2}$. The case of $\alpha = 1$ is simple (formally, it is a special case of Gabaix and Laibson (2006)):

competitors offer $q^1 = 2$ and $q^2 = 0$ in Nash equilibrium. The reasoning is simple: since the officer never considers dimension 2, competitors have no incentive to compete in this dimension, whereas competitive pressures along dimension 1 drive quality up in "Bertrand fashion", such that in equilibrium competitors earn zero profits.

The case of $\alpha \in [\frac{1}{2}, 1)$ is more interesting.

Proposition 4 *The game has a unique symmetric Nash equilibrium. In particular:*
(i) If $\alpha = \frac{1}{2}$, competitors play $q^1 + q^2 = 1$ with probability one and $q^1 \sim U[0,1]$.
(ii) If $\alpha \in (\frac{1}{2}, 1)$, competitors mix over average quality $c = \frac{1}{2}(q^1 + q^2)$ according to the cdf

$$G(c) = \frac{1-\alpha}{2\alpha - 1}\left[\frac{\alpha}{1-c} - 1\right]$$

defined over the interval $[1-\alpha, \alpha]$. The quality along each dimension is a deterministic function of c:

$$\begin{aligned} q^1 &= \frac{2\alpha}{2\alpha - 1}\left[c - (1-\alpha)\right] \\ q^2 &= \frac{2(1-\alpha)}{2\alpha - 1}\left[\alpha - c\right] \end{aligned}$$

Thus, when $\alpha \in (\frac{1}{2}, 1)$, competitors mix over average quality c in equilibrium. The greater the asymmetry in the dimensions' salience, the greater the range of values that c gets in equilibrium. The expectation of c is

$$E_G(c) = 1 - \frac{\alpha(1-\alpha)}{2\alpha - 1}\ln\left(\frac{\alpha}{1-\alpha}\right)$$

which is strictly increasing in α in the range $(\frac{1}{2}, 1)$. In equilibrium q^1 takes values in $[0, 2\alpha]$, while q^2 takes values in $[0, 2(1-\alpha)]$. The two components are linked deterministically by the linear equation

$$q^2 = 2(1-\alpha) - \frac{1-\alpha}{\alpha} q^1$$

The $\alpha \to \frac{1}{2}$ and $\alpha \to 1$ limits of this equilibrium characterization coincide with our analysis for these extreme cases. In particular, when $\alpha = \frac{1}{2}$, competitors plays $c = \frac{1}{2}$ with probability one, such that the support of the equilibrium strategy is a downward sloping line connecting the quality pairs $(0,1)$ and $(1,0)$.

The following corollary describes the competitors' equilibrium payoffs.

Corollary 1 *The firms' payoff in symmetric equilibrium is* $\alpha(1-\alpha)$.

To derive this result, consider the quality pair $(q^1, q^2) = (2\alpha, 0)$, which is an extreme point in the support of the equilibrium strategy. When a competitor plays this alternative, the officer chooses it if and only if he sampled dimension 1. Therefore, the competitor's equilibrium payoff is $\alpha \cdot [1 - \frac{1}{2}(0 + 2\alpha)] = \alpha(1-\alpha)$. Thus, equilibrium payoffs go down and average quality goes up when the dimensions' relative salience becomes more asymmetric. Thus, a stronger "salience bias" leads to a more competitive equilibrium outcome. The intuition is that when α approaches 1, the officer's choice becomes more predictable, and this strengthens competitive pressures among competitors. At the same time, more asymmetric salience also implies a greater variation in quality across dimensions, in the sense that the range of values that $|q^2 - q^1|$ becomes wider as α gets closer to 1.

Comment: Rational-choice interpretation

Recall that the DM's choice function in this example has a simple rational-choice interpretation: with probability α^k he genuinely cares about q^k only. From this perspective, it is not surprising that when discrimination is impossible, greater heterogeneity in DMs' preferences (captured by shifting α

toward $\frac{1}{2}$) results in a less competitive equilibrium outcome. For this interpretation to be valid, it is important to assume that the two quality dimensions are intrinsically bundled together, such that competitive pressures cannot lead to their "unbundling". At any rate, the rational-choice interpretation will break down in our next example.

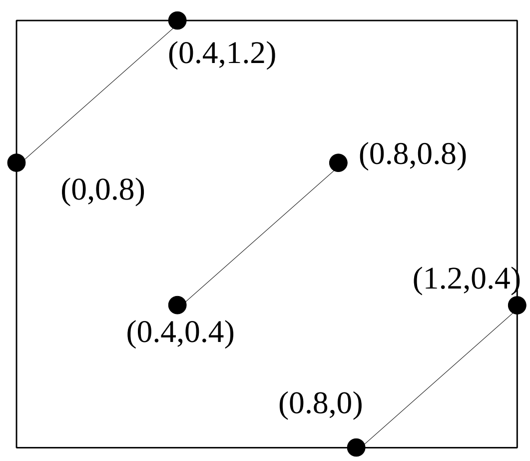
4.2 Choice Procrastination

Now suppose that the procurement officer follows the "choice procrastination" procedure described in Section 3. Specifically, when there is no dominant alternative, the officer chooses the outside/default option with probability $\lambda \in (0,1)$; and with the remaining probability $1-\lambda$, he resorts to the "sampling a dimension" rule with $\alpha = \frac{1}{2}$. As in the previous sub-section, we interpret the choice rule as the outcome of the officer's need to justify his actions to a superior. The difference is that now the officer has an additional tool for avoiding trade-offs - namely, he can refrain from making an explicit choice, which would intuitively make him less vulnerable to criticisms. Consequently, the officer may end up with the outside/default option even though it is a dominated alternative.

A different interpretation of the procedure does not involve the notion of justifiability, but relies on "Buridanic" indecisiveness. The officer realizes that the outside option is dominated by both market alternatives. However, he finds it difficult to choose between them because doing so requires him to perform trade-offs. When the officer makes an active choice, he chooses arbitrarily between the two undominated alternatives. However, he has a distaste for arbitrary choices, and this causes him to procrastinate, such that he may fail to reach a decision in the relevant time frame and end up with the dominated outside option, just like Buridan's Ass.

The officer's choice procedure in this example has the property that when one market alternative dominates the other, it is a dominant alternative altogether, and therefore chosen with probability one. In contrast, when neither market alternative dominates another, the market participation rate is only $1 - \lambda$. Therefore, by Proposition 3, domination must occur with positive probability in any symmetric Nash equilibrium.

Figure 1 represents the support of a symmetric Nash equilibrium strategy for $\lambda = \frac{1}{2}$. It consists of three 45-degree line segments. Each of these segments is realized with probability $\frac{1}{3}$. The distribution over average quality is given by some *cdf* over $[\frac{2}{5}, \frac{4}{5}]$, which is independent of the segment. Note that two quality pairs in the support dominate one another if and only if they belong to the same segment. Hence, the probability of domination is $\frac{1}{3}$. Therefore, the officer will end up with the outside option with probability $\frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$.



(Figure 1)

We do not have a full characterization of symmetric equilibria. However, we are able to characterize a family of equilibria of which the strategy illustrated by Figure 1 is an instance. For this purpose, we introduce some new notation. First, we represent a pure strategy (q^1, q^2) by the pair (p, e), where $2p = 2 - (q^1 + q^2)$ is the profit that the quality pair generates for the competitor conditional on being chosen, and $2e = q^1 - q^2$ is the quality variation the strategy exhibits. Second, for any positive integer n, denote

$$\sigma = \frac{1-\lambda}{2} \cdot \left(1 - \frac{1}{n}\right)$$

The interpretation of σ is simple: it is the probability that the officer chooses a "market alternative" when the probability that one market alternative dominates another is $\frac{1}{n}$.

Let $n \geq 3$ be an integer and let $d > 0$. Define $s^*(n,d)$ to be a mixed strategy that consists of *independent randomizations* over p and e, where:

(i) p is distributed according to the *cdf*

$$G(p) = (1+\sigma n)\left(1 - \frac{d\sigma n}{p}\right)$$

defined over the interval $[d\sigma n, d\sigma n + d]$.

(ii) e is uniformly distributed over the discrete set

$$\left\{d\left(k - \frac{1}{2}(n-1)\right)\right\}_{k=0,1,\ldots n-1}$$

The strategy illustrated in Figure 1 is thus $s^*(3, \frac{2}{5})$.

Proposition 5 *If* $n \in [1+\frac{1}{\lambda}, 1+\frac{3}{\lambda}]$ *and* $d = \frac{2}{\lambda+n(2-\lambda)}$*, then* $s^*(n,d)$ *is a symmetric Nash equilibrium strategy.*

Let us elaborate on the properties of this class of equilibrium strategies.

Structure of the support and domination probability. The support of the equilibrium strategy is divided into n line segments, which are vertical in the (p,e) representation (they have a 45° slope in (q^1,q^2) space, as in Figure 1). Each segment corresponds to a different value of e. The distance between adjacent segments is d, which is also the range of values that p can get. Therefore, two realizations $(p_1, e_1), (p_2, e_2)$ dominate one another if and only if $e_1 = e_2$. A larger n corresponds to an equilibrium with weaker comparability.

Participation rate. The structure of domination implies that the officer switches away from the outside option with probability $1 - \lambda + \lambda \cdot \frac{1}{n}$. The restriction on the values that n can get implies that the participation rate is bounded from above by

$$(1 - \lambda) + \lambda \cdot \frac{\lambda}{1 + \lambda} = \frac{1}{1 + \lambda}$$

Because n only gets integer values, these upper bounds are not tight. We will see below that the maximal participation rate overall is $\frac{1}{3}$ (obtained when λ is sufficiently large).

Quality and payoff. The marginal distributions over q^1 and q^2 are identical, with support $[0, dn]$. The upper bound of this interval is strictly above 1. The expected equilibrium average quality is

$$1 - d\sigma n(1 + \sigma n) \ln \left(\frac{1 + \sigma n}{\sigma n} \right)$$

Each competitor's expected payoff is

$$2d\sigma(\sigma n + 1)$$

It can be verified that the expected average quality is strictly greater than $\frac{1}{2}$, and the payoff is lower than $\frac{1}{2}$. That is, expected average quality is lower and the competitors' payoff is higher than when the officer has a vanishing propensity for default bias ($\lambda \to 0$). In this sense, when the officer's default bias is stronger, the equilibrium outcome more competitive.

Limit equilibria and welfare. As $\lambda \to 0$, the permissible values of n diverge, and the collection of line segments becomes infinitely dense, approximating the line $q^1 + q^2 = 1$, i.e. $p = \frac{1}{2}$ and $e \sim U\left[-\frac{1}{2}, \frac{1}{2}\right]$. The equilibrium participation rate thus converge to zero. On the other hand, there are two limit equilibrium distributions when $\lambda \to 1$ (i.e., when the officer has extreme propensity for default bias). In both of them, $q^1 + q^2 = 2$ with probability one, i.e. $p = 0$; in one of them, $n = 3$, such that e is uniformly distributed over $\{-\frac{1}{2}, 0, \frac{1}{2}\}$; while in the other, $n = 4$, such that e is uniformly distributed over $\{-\frac{3}{5}, -\frac{1}{5}, \frac{1}{5}, \frac{3}{5}\}$. Thus, the participation rate in this class of equilibria cannot exceed $\frac{1}{3}$. As usual, define the officer's welfare as the average quality he ends up getting. Then, his equilibrium welfare in the $\lambda \to 0$ limit is $\frac{1}{2}$. His maximal equilibrium welfare in the $\lambda \to 1$ limit is $\frac{1}{3}$ (because this is the maximal probability with which he switches away from the outside option, and then he enjoys an average quality of 1).

What is the broad intuition behind the relatively low upper bound $(\frac{1}{3})$ on the equilibrium probability of domination (in the class of equilibria we have isolated)? When λ is low, competitors have an incentive to lower the domination probability. They can do so by introducing large quality variation across attributes. On the other hand, when λ is high, competitors have an incentive to induce a high degree of comparability in order to attract the officer away from the outside option. However, comparability in this model is established by *domination*; and since the quality variation e can take values in a wide range, a competitor has to offer high quality along *both* dimensions in order to attain a substantial probability of domination. Thus, attaining domination is quite costly for the competitor (in terms of his payoff conditional on being chosen), and this restraints the incentive to increase comparability.

The class of equilibria under consideration turns out to be fully characterized by two properties that can be distilled from the above description. A mixed strategy s satisfies *independence* if it induces statistically independent distributions over p and e. We say that s satisfies *constant comparability* if $\Pr\{(p_1, e_1) \text{ dominates } (p_2, e_2) \mid (p_1, p_2)\}$ is the same for almost all (p_1, p_2), where (p_1, e_1) and (p_2, e_2) are two independent draws from s.

Proposition 6 *If a symmetric Nash equilibrium strategy satisfies independence and constant comparability, it must take the form* $s^*(n, d)$*, where* $n \in [1 + \frac{1}{\lambda}, 1 + \frac{3}{\lambda}]$ *and* $d = \frac{2}{\lambda + n(2-\lambda)}$.

The independence and constant-comparability properties are of interest because they provide a link to other models of competition under limited comparability. In Varian (1980), the fraction of consumers who make price comparisons is assumed to be an exogenous constant. In Piccione and Spiegler (2012), independence and constant comparability are logically linked by an underlying property of the comparability structure.

5 Default Architecture

So far, we have identified the outside option as a default alternative, such that if the DM does not make an active or explicit decision, he ends up with the outside option. In this section we relax this assumption and allow market alternatives to function as default options. We have two interpretations in mind. First, the competitive interaction may be recurrent (albeit with myopic agents, shutting down repeated-game effects), such that the DM already selected one of the competitors in a previous round. This competitor naturally serves as a default option in the current situation. Second, the default specification may be the result of deliberate "default architecture", as advocated by Thaler and Sunstein (2008). A regulator can assign the DM to a market alternative, such that the DM can switch away from it only if he explicitly requests so. The rationale for such an intervention is that it is likely to increase the market participation rate in the presence of default bias. However, this rationale does not take into account the competitors' equilibrium response to the intervention, which in turn relies on the psychology that generates default bias in first place. We will show that once equilibrium effects are taken into account, this type of default architecture can have a negative effect on the DM's welfare.

In general, when we relax the automatic identification of the outside option with the default, the DM's choice function s should be allowed to discriminate between choice problems in which the outside option is also the default and choice problems in which the default and outside options diverge. The general analysis of Section 3 becomes somewhat more complex as a result. In particular, it now makes sense to assume that a dominated market alternative is chosen with positive probability when it is the default - a situation that was ruled out before. However, when the outside option is $(0, 0)$ - as in the examples of the previous section and the present one - the analysis is unchanged, and our general results continue to hold. The remainder of this section is devoted to examples.

5.1 Price-Quality Competition

Suppose that the DM is a consumer who chooses whether to enter a market for some product. The competitors are firms that sell this product. The two considerations are the price and quality of any given alternative: q^1 represents quality, denoted q, whereas $2-q^2$ is the price, denoted p. Thus, firm i's payoff conditional on being chosen is $p_i - q_i$. The outside option is represented by the pair $q_0 = (0, 2)$. The interpretation is that the consumer has no external substitute for the traded product, such that he experiences zero quality and pays a zero price if he stays out. Note that the zero lower bound on q^2 translates into an upper bound of 2 on p. We define the consumer's "true welfare" from the alternative (p, q) to be $(1 + g)q_i - p_i$, where $g > 0$ is a constant that captures the gains from trade in this market.

The consumer follows the choice procedure of Section 4.2: if one alternative dominates all others, he chooses it; otherwise, he adheres to the default option with probability λ, and with the remaining probability $1-\lambda$ he chooses according to a randomly selected dimension (we assume that the two dimensions are equally likely to be sampled, and that ties with the outside option are resolved in its favor). Sampling dimensions has a natural interpretation in this context: either the consumer stops thinking about prices and chases quality, or he only thinks about prices, in which case he will choose not to participate in the market (unless a firm offers a negative price, an event that will not occur in equilibrium).

Let us consider two default rules. The first, known as "opt in", is what we have assumed so far - namely, the default is the outside option. The second, known as "opt out", is that the default option is a market alternative. To maintain the game's symmetry, we assume in this case that the DM is equally likely to be assigned to the two firms. In this context, "opt in" can be interpreted as a rule that outlaws automatic renewal of contracts: unless the consumer actively selects his old provider or switches to a different one, he ends up without a contract. In the same vein, "opt out" fits an environment in which all consumers have an existing provider, and the consumer's contract is renewed automatically unless he actively switches. We study symmetric Nash equilibrium under both default rules.

"Opt in"

Equilibrium analysis in this case is very simple. A firm can make non-negative profits only if it charges a non-negative price. Therefore, if the consumer samples the price dimension, he will always choose the outside option; the firm can be chosen only if the consumer samples the quality dimension. This means that firms effectively engage in Bertrand competition over quality (except that the sum of their market shares is equal to the probability that the consumer samples the quality dimension). Therefore, in symmetric Nash equilibrium firms choose $(p, q) = (2, 2)$ and earn zero profits. The consumer enters the market with probability $\frac{1}{2}(1 - \lambda)$, and therefore his true welfare in equilibrium is $g(1 - \lambda)$.

"Opt out"

Equilibrium analysis in this case is a bit more involved. As before, a firm can dominate the outside option only if it charges a non-positive price. Because the firm can ensure positive profits, it follows that domination will never occur with positive probability in equilibrium. Moreover, because the outside option will always outperform the market alternatives along the price dimension, firms have no incentive to charge a price below the maximum of 2. Therefore, in Nash equilibrium firms will play $p = 2$ with probability one. The only question is how they will randomize over quality.

Proposition 7 *Under the "opt out" default rule, there is a unique symmetric Nash equilibrium. Firms play $p = 2$ with probability one, and randomize over q according to the continuous, strictly increasing cdf*

$$F(q) = \frac{\lambda}{1-\lambda} \cdot \frac{q}{2-q}$$

over the interval

$$[0, 2(1-\lambda)]$$

Each firm earns a profit of λ.

The equilibrium structure is a variant on Varian (1980). Firms compete along the quality dimension alone. When $q_i < q_j$, firm i's alternative is dominated, and therefore chosen only because of default bias, hence the size of its clientele is $\frac{1}{2}\lambda$. If a firm offers a quality arbitrarily close to $q = 0$, then it will earn a payoff of $(2-0) \cdot \frac{1}{2}\lambda = \lambda$. The realization $q = 0$ is in the support of the equilibrium strategy. When a firm offers higher quality, it increases its clientele because it becomes undominated with higher probability and therefore attracts clientele from the other firm with positive probability.

We are now able to compute the implications of the switch from "opt in" to "opt out" for consumer welfare. Under "opt out", the consumer's equilibrium participation rate is $1 - \frac{1}{2}(1-\lambda) = \frac{1}{2}(1+\lambda)$ for every realization of the firms' strategies (ignoring the zero-probability event that $q_1 = q_2$). If firms chose quality competitively, they would offer $q = p = 2$ and earn zero profits, such that the consumer's utility conditional on entering would be $2g$. Because each firm earns a profit of λ in equilibrium, the consumer's net equilibrium welfare is

$$\frac{1}{2}(1+\lambda)\cdot 2g - 2\lambda = g + \lambda g - 2\lambda$$

This quantity can be negative, such that the consumer is worse off on average relative to the outside option.

We saw earlier that the consumer's equilibrium welfare under "opt in" is $g(1-\lambda)$. It follows that the switch from "opt in" to "opt out" increases the consumer's net equilibrium welfare if and only if $g > 1$. Thus, as long as the gain from trade is not too large, the switch harms the consumer. The reason is that the "opt in" rule encourages firms to choose quality competitively. In contrast, the "opt out" rule increases their effective market power because they enjoy monopoly power over consumers who are initially assigned to them and succumb to the Buridanic fallacy. As a result, the equilibrium effect of the switch from "opt in" to "opt out" is that firms offer lower quality on average. When the gross gains from trade are not too large, the equilibrium effect more than offsets the benefit from higher market participation.

5.2 Revisiting the "Procurement Officer" Example

Recall the example of Section 4.2, and modify the default rule into "opt out" - that is, one of the two service providers is pre-specified (with probability $\frac{1}{2}$) as a default option for the procurement officer. Because the outside option is always dominated by the two market alternatives, the officer will never opt out - he will either cling to the default provider or switch to another. This means that when neither market alternative dominates the other, the officer chooses each of them with probability $\frac{1}{2}$, such that this example is reduced to the model of Section 4.1 with $\alpha = \frac{1}{2}$. The analysis of default architecture thus becomes a comparison of equilibria in Sections 4.1 and 4.2.

Let us use the average quality of the officer's selected alternative as a measure of his true welfare. In the unique symmetric equilibrium of Section 4.1, both competitors offer an average quality of 1 with probability one, and the officer chooses one of them. Therefore, his equilibrium welfare is 1. In contrast, in Section 4.2 we saw that there are multiple equilibria, and we were able to characterize a certain class of equilibria that are parameterized by the constant n (the number of parallel 45-degree segments that constitute the support of the equilibrium strategy). In such an equilibrium, the officer's welfare is given by the market participation rate minus industry profits (because if firms offered only zero-profit quality pairs, the officer's payoff from entering the market would be equal to 1) :

$$1-\lambda+\frac{\lambda}{n}-2d\sigma(\sigma n+1)=\frac{(1-\lambda)\,n^2+\lambda n+1}{(2-\lambda)\,n^2+\lambda n}$$

Maximal consumer welfare (attained at the minimal permissible n for any given λ) is in the interval $(\frac{1}{3},\frac{1}{2})$ and decreasing in λ almost everywhere.

Thus, given the set of equilibria we have focused on under "opt in", this default regime is inferior to "opt out" in terms of the officer's welfare: the increase in the participation rate thanks to the switch from "opt in" to "opt out" outweighs the decrease in the quality of alternatives that competitors offer in equilibrium. The two forces were also at play in the example of Section 5.1, but their net effect was different. We conclude that while the adverse equilibrium effect of the switch from "opt in" to "opt out" is robust, whether it outweighs the beneficial effect on "market participation" depends on the specifics of the model.

5.3 Discussion

The observation that people tend to choose by default in difficult choice situations implies that "default architecture" can have dramatic implications for eventual choice patterns. This has led researchers (e.g. Thaler and Sunstein (2008), Beshears et al. (2012)) to advocate switching from "opt in" default regimes to "opt-out" or "no default" (i.e. active choice) rules, in settings such as retirement savings by employees. It should be emphasized that in these settings, funds do *not* compete spontaneously and directly over employees; instead, contracts are mediated by employers. In other words, these markets are regulated de facto, except that the regulator is not the government but the employer.

The modeling exercise in this paper helps clarifying the respective roles of this de-facto regulation and default architecture in such settings, and speculating about the equilibrium effects of redesigning defaults if employers ceased to mediate the interaction between funds and savers, and spontaneous and direct competition among funds over savers *were* the norm. What would be the consequences of default architecture in this case? Our analysis suggests that a shift from "opt in" to "opt out" or "no default" would indeed raise the overall level of participation in retirement saving programs; however, at the same time it would raise the management fees that funds charge. The magnitude of this effect will be large if the population of savers is relatively "indecisive". The intuition is that the opt-out rule gives funds greater effective market power; because they benefit from default bias, they have a stronger incentive to induce it by creating difficult choices for savers.

A broad lesson from our exercise is that when we wish to analyze regulatory interventions that address consumer decision errors, it is important to have an explicit procedural model of consumer choice, which provides a concrete "story" behind the consumers' errors, and enables us to speculate about the market equilibrium's response to the intervention. For further exploration of this theme, see Spiegler (2015,2017).

6 Concluding Remarks

This paper explored the theoretical implications of trade-off avoidance for competitive behavior in the simplest possible environment, in which two agents compete in two-dimensional objects for a single DM, such that if the DM were conventionally rational the interaction would collapse to standard Bertrand competition. We assumed instead that the DM follows a (probabilistic) non-compensatory choice procedure, which may exhibit default bias (in the sense that the DM may choose a dominated default alternative in difficult choice situations). We saw that trade-off avoidance leads to non-competitive equilibrium outcomes.

In applications, the departure from the competitive benchmark was larger when the DM's default bias was weaker and when his procedure tended to treat the two dimensions more symmetrically. Thus, aspects of the choice procedure that are intuitively viewed as biases (a tendency to choose the default even when there is no good reason for doing so, asymmetric salience of dimensions) end up "protecting" the DM, in the sense that they encourage more competitive equilibrium behavior by market agents. By the same token, regulatory interventions that attempt to overcome these biases may have counterproductive equilibrium effects.

More than two dimensions

We can extend the model to n dimensions such that each competitor $i = 1,2$ offers an alternative characterized by a vector $q_i \in \mathbb{R}^n_+$. Competitor i's payoff given the strategy profile (q_1, q_2) is $(n - \sum_{k=1}^{n} q_i^k) \cdot s_i(q_0, q_1, q_2)$. While thorough generalization of our analysis is outside the scope of this paper, the following is an example of how one of the main results can be extended. Consider the "*sampling a dimension*" procedure where in the absence of a dominant alternative, the DM focuses on one dimension at random and selects the best alternative along this dimension (with arbitrary tie breaking). Then, it can be shown that in any symmetric Nash equilibrium, $q \not> q'$ for every two realizations q, q' of the equilibrium strategy that belong to same hyperoctant relative to q_0. The proof, which extends the arguments in the proof of Proposition 1, is available upon request.

Related Literature

This paper is the first to analyze competition for "trade-off averse" DMs who follow an entirely non-compensatory choice procedure. In an independent paper, Papi (2014b) analyzes a model in which consumers are limited in the number of attributes they can trade off. When market products contain more attributes, consumers focus on a subset which is a function of firms' marketing strategies. Thus, one key difference between the two works is that in our model, consumer choice is entirely based on ordinal rankings, whereas in Papi's model, consumers maximize a continuous utility function over a restricted set of attributes. Papi (2014a) axiomatizes a choice procedure that mixes compensatory and non-compensatory elements, and applies it to a Stackelberg model. In his model, the DM uses a non-compensatory procedure only to shrink the choice set into a small "consideration set", to which he applies a well-defined utility function. Bachi (2014) studies uni-dimensional price competition when consumers are unable to perceive small price differences.

The choice theory literature has studied models with a non-compensatory component. Rubinstein (1988) analyzed a choice procedure related to ours, where the DM regards one alternative in $\mathbb{R}^2$ as dominating another if it is "approximately the same" along one dimension and significantly better along the other. Mariotti and Manzini (2007) axiomatized a "sequentially rationalizable" choice procedure that employs a succession of binary relations to eliminate alternatives from the choice set.

Choice models in which the DM tends to stick to a status-quo / default alternative when facing "too little dominance" have been studied by Masatlioglu and Ok (2005,2014) and Teper and Riella (2014), among others – extending a tradition of multi-utility representations of incomplete preferences due to Bewley (1986) and Ok (2002). A key feature in Masatlioglu and Ok's axiomatizations is that they examine choice problems with and without a default; this mirrors the distinction between the "opt in" and "no default" regimes in our model. Dean (2008) conducted an experimental test of axioms that characterize various families of models of decision avoidance. Finally, the randomness in our DM's response to "hard choices" (which results from the "decisiveness" typology) links it to recent models of stochastic choice, where randomness derives from new primitives (e.g., consideration sets in Manzini and Mariotti (2014)), unlike conventional random-utility models.

Another strand in the literature (reviewed in Farrell and Klemperer (2007)) studies multi-period models in which the DM may stick an already-chosen alternative in subsequent periods due to costly switching. Ericson (2016) extends it to study the welfare implications of various default regimes when DMs do not know their exact future switching costs, under both monopoly and competitive equilibrium settings. Carroll et al. (2009) analyze a non-equilibrium model in which the DM exhibits a default bias due to quasi-hyperbolic preferences in the context of retirement savings plans.

The market interpretation of our model links it to the "behavioral industrial organization" literature (see Spiegler (2011) for a textbook treatment). Within this literature, two papers are most closely related. Gabaix and Laibson (2006) analyze a model in which two firms compete in price pairs, where a fraction of consumers are unaware of dimension 2, and thus choose purely on the basis of price rankings along dimension 1, while the remaining consumers are conventionally rational and choose the firm with the lowest true price. Consumers have an outside option, the value of which is correlated with their type (the interpretation is that more sophisticated consumers are more likely to find a good outside option). Indeed, the "no default" version of our model can be interpreted in terms of unawareness: the DM focuses on one dimension because he is unaware of the other. An aspect of Gabaix and Laibson (2006) which is not dealt with in this paper is the endogeneity of attribute salience due to firms' disclosure decisions.

Spiegler (2006) analyzes a model in which n firms choose price *cdf*s over $(-\infty, 1]$. A firm's profit conditional on being chosen is the expected price according to its own *cdf*. The consumer chooses by taking a sample point from each of the *cdf*s and selecting the cheapest firm in his sample. As Spiegler (2006) notes, this can be viewed as a reduced form of a model in which firms choose infinite-dimensional price vectors and the consumer chooses according to the price ranking in a randomly selected dimension. This interpretation forms a clear link with the present model, and suggests an interesting generalization of our model to the case of n firms and K dimensions, in which consumers choose according to some probabilistic aggregation of the ordinal rankings along each dimension. From this perspective, Spiegler (2006) assumes a specific aggregation rule – random dictatorship – and takes the limit $K \to \infty$.[2]

Piccione and Spiegler (2012) present an alternative approach to modeling market competition under limited comparability. A market alternative consists of a "real price" and a "price format", and consumers can make a price comparison if and only if the two firms' price formats are comparable, according to some exogenous comparability structure. Carlin (2009) and Chioveanu and Zhou (2013) study special cases of this limited comparability formalism and extend them to the many-firms case. All these papers can be viewed as extensions of Varian (1980), who studied price competition when an exogenous fraction of the consumer population does not make comparisons. The new models essentially endogenize this parameter as a consequence of the firms equilibrium obfuscation tactics. Piccione and Spiegler (2012) assume that consumers have no outside option. Following the example of the present paper, Spiegler (2015) extends the Piccione-Spiegler model to incorporate an outside option, and performs a rudimentary comparison between the "opt

[2] From a technical point of view, this general problem has affinities with the majority auction games studied by Szentes and Rosenthal (2003).

in" and "opt out" default rules.

Finally, the role of attribute salience was studied by Koszegi and Szeidl (2013) and Bordalo et al. (2013a). These papers model salience as a systematic distortion of decision weights, while the present paper captures the salience of an attribute by the probability it is considered by a trade-off avoiding DM. Bordalo et al. (2013b) and Spiegler (2013) analyze market models in which consumers' decision weights are endogenously determined by firms' pricing and marketing equilibrium strategies.

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Appendix: Proofs

Proposition 1

First, note that competitors must earn strictly positive profits in Nash equilibrium, because each competitor can secure a strictly positive profit by playing a mixed strategy with full support over $\{q \mid \sum_{k=1}^{2} q^k \leq 2\}$. Let us now introduce some useful notation. A symmetric mixed strategy equilibrium is a probability measure μ over the set $\{q \in \mathbb{R}_+^2 \mid \sum_{k=1}^{2} q^k \leq 2\}$. Let F^k denote the marginal *cdf* over q^k induced by μ. That is,

$$F^k(q^k) = \int_{q^{-k}} \int_{r^k \leq q^k} d\mu.$$

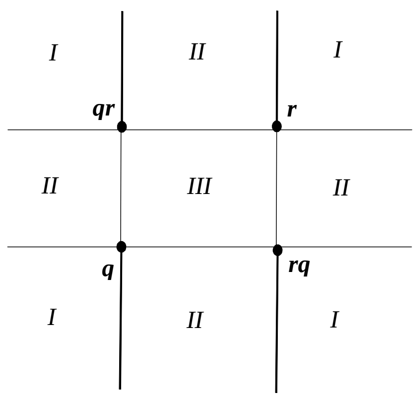
Let F^{k-} be the left limit of F^k, i.e. $F^{k-}(x) = \lim_{y \to x^-} F^k(y)$. Finally, in what follows, O denotes some quadrant w.r.t q_0, which is closed from below in both dimensions. Note that at most three quadrants are relevant: competitors will never play $q < q_0$ in equilibrium (the reason is that by assumption, a market alternative that is dominated by q_0 is never chosen, and therefore generates zero profits).

Fix the outside option $q_0 \in \mathbb{R}^2$ and let us first establish that μ is continuous, that is, $F^{k-} \equiv F^k$, $k = 1, 2$. Assume, in contradiction, that w.l.o.g F^1 contains an atom on some q^1. Consider the lowest q^2 such that (q^1, q^2) is in $Supp(\mu)$. If $\sum_{k=1}^{2} q^k = 2$, then competitors make zero profits in equilibrium, a contradiction. If $\sum_{k=1}^{2} q^k < 2$, a conventional "undercutting" argument applies: if a competitor deviates to $(q^1 + \varepsilon, q^2)$, where $\varepsilon > 0$ is arbitrarily small, then by strict monotonicity the increase in the competitor's probability of being chosen outweighs his loss in profit conditional on being chosen.

Next, we show that if F^k contains a "hole", then it is an interval $\left[a^k, q_0^k\right]$, for $k = 1, 2$. Assume the contrary. W.l.o.g, let $[a^1, b^1]$ be an interval such that $F^1(q^1) = c$ for any $q^1 \in [a^1, b^1]$, $F^1(b^1 + \varepsilon) > c$ for some arbitrary small ε, and both a^1 and $b^1 + \varepsilon$ have the same ordinal position relative to q_0^1. Let $(b^1, q^2) \in Supp(\mu)$. If a competitor deviates to $(b^1 - \delta, q^2)$ where $\delta > 0$ is small, the competitor's probability of being chosen does not change, but his profit conditional on being chosen increases, hence the deviation is profitable.

Let $q, r \in Supp(\mu) \cap O$ for some quadrant O, such that $r > q$. Denote $qr = (q^1, r^2)$, $rq = (r^1, q^2)$. We will show that a deviation to qr or rq is profitable. Denote a competitor's payoff conditional on being chosen when playing w by $P(w)$. Observe that $P(q) + P(r) = P(rq) + P(qr)$.

Assume the opponent plays some $t \in Supp(\mu)$ (not necessarily in O). Because μ is continuous, $t^k \notin \{q^k, r^k, q_0^k\}$ with probability one, for both $k = 1, 2$. The ordinal position of t relative to the four points q, r, qr, rq can be divided into three cases, denoted I, II and III, and described by Figure 2:



(Figure 2)

In case I, t does not lie between the four points along any dimension. In this case, all four points generate the same probability of being chosen when played against t. In case II, t lies between the four points in exactly one dimension. In this case, the two points that lie on the same side of t along that dimension generate the same probability of being chosen when played against t. In case III, t lies between the four points along both dimensions - i.e., it is in $co\{q, r, rq, rq\}$. In this case, the four points may generate different probabilities of being chosen when played against t.

For each $w \in \{q, r, qr, rq\}$, denote $s_w = s_1(q_0, w, t)$. Let us now consider a competitor's expected payoff from playing w against t, according to the three cases.

Case I. As we saw, $s_w = s_{w'}$ for all $w, w' \in \{q, r, qr, rq\}$. Clearly, this implies

$$s_q P(q) + s_r P(r) = s_{rq} P(rq) + s_{qr} P(qr)$$

Case II. W.l.o.g, assume $s_q = s_{qr}$ and $s_{rq} = s_r$. Note that $s_q < s_r$ and $P(q) > P(qr)$, which implies

$$\begin{aligned} s_q P(q) + s_r P(r) &= s_q P(q) + s_r (P(rq) + P(qr) - P(q)) \\ &= (s_q - s_r)(P(q) - P(qr)) + s_q P(qr) + s_r P(rq) \\ &< s_q P(qr) + s_r P(rq) = s_{rq} P(rq) + s_{qr} P(qr) \end{aligned}$$

Case III. Here, t is in the convex hull of $\{q, r, qr, rq\}$. By the assumption that domination of one market alternative by another does not increase the market participation rate, $s_q + s_r \leq s_{qr} + s_{rq}$. By strict monotonicity $s_q < s_{rq}$ and $s_q < s_{qr}$. Since $P(rq) > P(r)$ and $P(qr) > P(r)$ we have

$$\begin{aligned} s_q P(q) + s_r P(r) &= s_q (P(qr) + P(rq) - P(r)) + s_r P(r) \\ &= s_q (P(qr) - P(r)) + s_q (P(rq) - P(r)) + (s_q + s_r) P(r) \\ &< s_{qr} (P(qr) - P(r)) + s_{rq} (P(rq) - P(r)) + (s_{qr} + s_{rq}) P(r) \\ &= s_{qr} P(qr) + s_{rq} P(rq) \end{aligned}$$

Note that case I cannot occur with probability one - otherwise, q would yield a higher payoff than r. Thus, by integrating over all possible t according to μ, we can conclude that the sum of the payoffs at q and r is lower than the sum of the payoffs at qr and rq. Therefore, at least one of the points qr and rq are profitable deviations, a contradiction.

Proposition 2

Let μ be a symmetric equilibrium strategy. Because $q_0 = (0, 0)$, market alternatives necessarily belong to same quadrant relative to q_0. Therefore, Proposition 1 implies that the probability that two draws from μ dominate one another is zero. The claim then immediately follows from the continuity and "no holes" properties derived in the proof of Proposition 1.

Proposition 3

Assume in contradiction that there exists a symmetric Nash equilibrium in which domination of one market alternative by another occurs with zero probability.

Let us first establish that μ is continuous. First, suppose μ contains an atom at some q. Then, by the assumption that domination of one market alternative by another increases the market participation rate, a deviation to $q + (\varepsilon, \varepsilon)$ for some arbitrarily small $\varepsilon > 0$ is profitable. Second, suppose that μ contains a singular atom - w.l.o.g, F^1 has an atom on some q^1 and yet there is no q^2 such that μ contains an atom on (q^1, q^2). This possibility is inconsistent with the assumption that domination of one market alternative by another occurs with zero probability in equilibrium, a contradiction. It follows that F^1 and F^2 are continuous.

Let O be some quadrant relative to q_0 such that $Supp(\mu) \cap O \neq \varnothing$, and let $q, r \in Supp(\mu) \cap O$. As in the proof of Proposition 1, denote $qr = (q^1, r^2)$, $rq = (r^1, q^2)$. By the preceding step, we can assume w.l.o.g $q^1 < r^1$, $q^2 > r^2$, such that $[q^1, r^1] \subset Supp(F^1)$ and $[r^2, q^2] \subset Supp(F^2)$. Furthermore, any realization t of the opponent's strategy must fall into one of the following three regions: (i) $t \in co\{q, r, qr, rq\}$; (ii) $t^1 < q^1$ and $t^2 > q^2$; (iii) $t^1 > r^1$ and $t^2 < r^2$. Note that if t is in regions (ii) or (iii), then each of the four points q, r, qr, rq generates the same probability of being chosen against t. Accordingly, let A denote the probability that competitor j is chosen conditional on the event that $q_i \in \{q, r, qr, rq\}$ and $q_j \notin co\{q, r, qr, rq\}$.

Denote $\alpha = s_1(q_0, qr, rq)$, $\beta = s_1(q_0, r, q)$, $\gamma = s_2(q_0, r, q)$, $\varepsilon = \alpha - \beta - \gamma$.

Note that by assumption, $\varepsilon > 0$. The payoff from the strategy q is

$$\left[2 - (q^1 + q^2)\right] \cdot \left[A + \gamma \left(F^1\left(r^1\right) - F^1\left(q^1\right)\right)\right]$$

and the payoff from r is

$$\left[2 - (r^1 + r^2)\right] \cdot \left[A + \beta \left(F^1\left(r^1\right) - F^1\left(q^1\right)\right)\right]$$

Because $q, r \in Supp(\mu)$, these payoffs are identical.

If a competitor deviates to qr, his payoff will be

$$\left[2 - (q^1 + r^2)\right] \cdot [A]$$

On the other hand, if the competitor deviates to rq, his payoff will be

$$\left[2 - (r^1 + q^2)\right] \cdot \left[A + \alpha \left(F^1(r^1) - F^1(q^1)\right)\right]$$

For μ to be an equilibrium, both expressions must be weakly below the payoff at q. Denote $B = F^1\left(r^1\right) - F^1\left(q^1\right)$. Then, the payoffs at the four points q, r, qr and rq can be written as follows:

$$\begin{aligned}
\pi\left(q\right) &= \left[2 - (q^1 + q^2)\right] \cdot [A + \gamma B] \\
\pi\left(r\right) &= \left[2 - (r^1 + r^2)\right] \cdot [A + \beta B] \\
\pi\left(qr\right) &= \left[2 - (q^1 + r^2)\right] \cdot [A] \\
\pi\left(rq\right) &= \left[2 - (r^1 + q^2)\right] \cdot [A + \left(\beta + \gamma - \delta + \varepsilon\right) B]
\end{aligned}$$

It follows that

$$\begin{aligned}
& \pi(p) + \pi(q) - \pi\left(qr\right) - \pi\left(rq\right) \\
= \;& B\left[\left(2 - \left(r^1 + q^2\right)\right)\left(\gamma + \beta - \alpha\right) + \gamma\left(r^1 - q^1\right) + \beta\left(q^2 - r^2\right)\right]
\end{aligned} \tag{1}$$

Note that $B > 0$. If q and r are sufficiently close, expression (1) is strictly negative, which means that the deviation to rq or qr is profitable, a contradiction.

Proposition 4

By Proposition 2, we can describe the support of μ by a continuous and strictly decreasing function $g : [0, \bar{q}^1] \to [0, \bar{q}^2]$, where for each q^1 in the support of F^1, $g(q^1)$ is the unique q^2 for which (q^1, q^2) is in the support of μ. Therefore,

$$F^2\left(g\left(q^1\right)\right) = 1 - F^1\left(q^1\right) \tag{2}$$

for every $q^1 \in [0, \bar{q}^1]$. Since F^1 and F^2 are strictly increasing, they are differentiable almost everywhere, such that the slope of g is

$$g' = -\frac{dF\left(q^1\right)/dq^1}{dF^2\left(q^2\right)/dq^2} \tag{3}$$

for almost every (q^1, q^2) along the graph of g.

Let us now write down an individual competitor's payoff function when the opponent plays μ:

$$\pi\left(q^1, q^2\right) = \left[2 - q^1 - q^2\right]\left[\alpha^1 F^1(q^1) + \alpha^2 F^2(q^2)\right]$$

In equilibrium, first-order conditions must hold. Thus, for both $k = 1, 2$, the equation

$$[\alpha^1 F^1(q^1) + \alpha^2 F^2(q^2)] = \alpha^k \cdot \frac{dF^k(q^k)}{dq^k} \cdot (2 - q^1 - q^2) \tag{4}$$

must hold almost everywhere along the graph of g. The L.H.S of the equations for $k = 1$ and $k = 2$ are identical, and so we obtain

$$\frac{dF^1\left(q^1\right)/dq^1}{dF^2\left(q^2\right)/dq^2} = \frac{\alpha^2}{\alpha^1}$$

By (3), we conclude that

$$g'\left(q^1\right) = -\frac{\alpha^2}{\alpha^1}$$

almost everywhere along the graph of g. Therefore, we can write g as follows:

$$g(q^1) = \bar{q}^2 - \frac{\alpha^2}{\alpha^1} q^1 \tag{5}$$

Let us now distinguish between two cases.

(i) $\alpha^1 = \frac{1}{2}$. Then, $g' = -1$. This means that $\bar{q}^1 = \bar{q}^2 = q^1 + q^2 = c$ for every (q^1, q^2) in the support of μ, where c is some constant we need to derive. In other words, the support of μ is a straight line that connects the points $(0, c)$ and $(c, 0)$. Plug (2) into (4) and obtain the simplified equation

$$1 = \frac{dF^k\left(q^k\right)}{dq^k} \cdot (2 - q^1 - q^2)$$

Because $q^1 + q^2 = c$ throughout the support of μ, $dF^k\left(q^k\right)/dq^k$ is constant as well:

$$\frac{dF^k\left(q^k\right)}{dq^k} = \frac{1}{2-c}$$

But this also means that q^k is distributed uniformly over $[0, c]$. Therefore, $c = 1$, which pins down the characterization.

(ii) $\alpha^1 \in (\frac{1}{2}, 1)$. The two extreme points in the support, $(0, \bar{q}^2)$ and $(\bar{q}^1, 0)$, must both generate the equilibrium payoff:

$$\alpha^1 \cdot (2 - \bar{q}^1) = \alpha^2 \cdot (2 - \bar{q}^2) = \pi \tag{6}$$

The two points are also linked by (5), if we plug $g(\bar{q}^1) = 0$. Combining these two equations, we obtain a solution for $\bar{q}^1$, $\bar{q}^2$ and for the equilibrium payoff π. Moreover, according to (5), every realization of total cost $q^1 + q^2$ in this interval is associated with a unique (q^1, q^2), as given in the statement of the proposition. Let us derive F^1. Since every (q^1, q^2) in the support of μ must be a best-reply, we must have that for every $q^1 \in [0, \bar{q}^1]$:

$$\left[2 - q^1 + g(q^1)\right] \cdot \left[\alpha^1 F^1\left(q^1\right) + \alpha^2 F^2\left(g(q^1)\right)\right] = \pi \tag{7}$$

Since every q^1 is associated with a unique average quality $c(q^1) = \frac{1}{2}[q^1 + g\left(q^1\right)]$ that increases with q^1, $F^1(q^1) = G(c(q^1))$, where G is the induced *cdf* over c. Plugging (2) and (5) into (7), we obtain an explicit expression for F^1 over $[0, \bar{q}^1]$, and hence also for G:

$$G(c) = \frac{1}{2\alpha - 1}\left[\frac{\pi}{1 - c} - (1 - \alpha)\right]$$

Because $c = \frac{1}{2}(q^1 + q^2)$, By (5) and (6), $\bar{q}^1 = 2\alpha$ and $\bar{q}^2 = 2(1 - \alpha)$, such that the equilibrium payoff is $\pi = \alpha(1 - \alpha)$. This pins down G and g, hence the values that c can get, as well as the values of (q^1, q^2) as a function of c.

The last step (for both cases) is checking that there are no profitable deviations. It suffices to consider deviations to pure strategies $(q^1, q^2) \in [0, \bar{q}^1] \times [0, \bar{q}^2]$. It is easy to verify that given the explicit expressions for F^1 and F^2, the payoff function

$$\pi\left(q^1, q^2\right) = \left[2 - q^1 - q^2\right]\left[\alpha^1 F^1(q^1) + \alpha^2 F^2(q^2)\right]$$

is decreasing (increasing) in both arguments when (q^1, q^2) is above (below) the graph of g, hence the maximal payoff is obtained at the points along g.

Proposition 5

Let $n \in [1 + \frac{1}{\lambda}, 1 + \frac{3}{\lambda}]$ and $d = \frac{2}{\lambda + n(2 - \lambda)}$. The strategy $s^*(n, d)$ induces a marginal distribution over q^k, with support $[0, \bar{q}^k]$ where $\bar{q}^k = nd$, $k = 1, 2$. It is easy to verify that each point on the support of $s^*(n, d)$ yields the same payoff. Clearly, when we look for profitable deviations from $s^*(n, d)$, we need only look for pure strategies $(q^1, q^2) \in [0, \bar{q}^1] \times [0, \bar{q}^2]$. From now on, we adhere to the (p, e) representation of strategies. We index the n values that

e by $k = 0, 1, ..., n-1$. Let $l = d\sigma n$ and $h = d(\sigma n + 1)$ denote the lowest and highest values in the support of the marginal distribution over p. Define $L^k = \{(p, e) \mid p \in [l, h] \text{ and } e = e^k\}$. That is, L^k is one of the n line segments that constitute the support of $s^*(n, d)$, which is associated with e^k. There are three cases to consider.

Case 1: Deviation to (p, e) where $p \geq h$.
For any $p \geq h$, it suffices to look for the most profitable deviation (p, e). The fact that e is uniformly distributed over evenly spaced values independently of p, and that $d = h - l$ and $d = e^k - e^{k-1}$, the total length of the $\{L^k\}$ segments that (p, e) is dominated by is independent of e. Moreover, the number of segments that partially dominate (p, e) is at most 2. Because of the concavity of G, it is more profitable to be partially dominated by one segment (the dominating prices on that segment being $[l, l + x + y]$ for some x and y) than being partially dominated by two segments (where the dominating prices are $[l, l + x]$ and $[l, l + y]$). This implies that for a given p the most profitable e maximizes the number of line segments L^k that entirely dominate (p, e). Therefore, in the sequel we restrict attention w.l.o.g to $e = 1 - p$, i.e., to $(0, q)$, where $q < 1 - h$, in the (q^1, q^2) representation.

Consider a deviation to $p = h + (m + x)\frac{d}{2}$, $m = 0, 1, .., n-2$, $x \in [0, 1]$. The payoff is

$$\left(h + (m + x)\frac{d}{2}\right)\frac{1-\lambda}{2}\left(1 - \frac{1}{n}\left(1 + m + G\left(l + dx\right)\right)\right)$$

Note that for $x = 0$, the payoff at $m = 0$ (which corresponds to no deviation) is higher than at $m = 1$ if and only if $n \leq 1 + \frac{3}{\lambda}$. Second, if this is the case, then the payoff continues to decrease for any $m > 1$ ($n \leq 1 + \frac{3}{\lambda}$ is a sufficient condition for the derivative of the payoff w.r.t m is negative for $m > 1$ and $x = 0$).

Moreover, the derivative of the payoff function w.r.t x (for a given m) is increasing. Thus, for each m, the maximal payoff is achieved at $x \in$

$\{0,1\}$.This, together with the previous result, imply that deviations to $p \geq h$ are unprofitable if and only if $n \leq 1 + \frac{3}{\lambda}$

Case 2: Deviation to (p,e) where $p \leq l$.
By the same argument as in Case 1, the most profitable deviation for a given $p \leq l$ is to e that maximizes the number of entire segments L^k which are dominated by (p,e). Therefore, in the sequel we restrict attention w.l.o.g to $e = (l + e^0) - p$, i.e., to $(\bar{q}^1, q)$, where $q > 1 - l$, in the (q^1, q^2) representation.

Consider a deviation to $p = l - (m+x)\frac{d}{2}$, $m = 1,..,n-2$, $x \in [0,1]$. The payoff is:

$$\left(l - (m+x)\frac{d}{2}\right)\left(\frac{1-\lambda}{2} + \frac{1+\lambda}{2}\frac{1}{n}\left(1 + m + \left(1 - G\left(h - dx\right)\right)\right)\right)$$

Note that the payoff at $m = 0$ (corresponding to no deviation) is higher than at $m = 1$ if and only if $n \leq 1 + \frac{3-\lambda}{(1-\lambda)\lambda}$. Second, if this is the case, then the payoff continues to decrease for any $m > 1$. Note that $n \leq 1 + \frac{3}{\lambda}$ implies $n \leq 1 + \frac{3-\lambda}{(1-\lambda)\lambda}$.

The derivative of this function w.r.t x implies the following: (i) it is increasing in x for $m \leq \sigma n - 1$; (ii) it is negative for $m > \sigma n - 1$. Thus, it is enough to check for deviation to $x = 0$ and $m \leq \sigma n - 1$, and by the previous result, these deviation are unprofitable for $n \leq 1 + \frac{3}{\lambda}$.

Case 3: Deviation to (p,e) where $l \leq p \leq h$.
Fix $p \in [l,h]$. Because any (p,e) where p is in this interval is comparable to points in at most 2 segments, and because all segments have the same probability distribution, it is enough to check for deviations from (p, e^0) to $(p, e^0 + x)$, where $x \in \left(0, \frac{d}{2}\right)$. Thus, $(p, e^0 + x)$ is comparable only to points on L^0 and L^1. Consider these three cases:

(i) $p + x \leq h$ and $p - x \geq l$. In this case $(p, e^0 + x)$ is not dominating, nor being dominated by, any point in L^1. As x increases, $(p, e^0 + x)$ is dominated by less points on L^0 but also dominates less. The competitor's net gain of market share is

$$\frac{1}{n}\frac{1-\lambda}{2}\left[G(p)-G(p-x)\right]-\frac{1}{n}\frac{1+\lambda}{2}\left[G(p+x)-G(p)\right]$$

Substituting G, we obtain the following condition for the deviation's profitability:

$$\frac{p+x}{p-x}>\frac{1+\lambda}{1-\lambda}$$

It is easy to verify that the L.H.S is maximized at $p=l+\frac{d}{2}$ and $x=\frac{d}{2}$, and the inequality is satisfies iff $n<1+\frac{1}{\lambda}$.

(ii) $p+x>h$. In this case (p,e^0+x) is dominated by some prices in L^0 and in L^1,but not dominating any point. Because the total length of the segments of L^0 and L^1 that dominate (p,e^0+x) is constant for any such x, the concavity of G implies that it is more profitable to be dominated by L^0 alone than by both. That is, this deviation is strictly less profitable than the deviation to (p,e^0+h-p) which is covered in case *(i)*.

(iii) $p-x<l$. In this case (p,e^0+x) is dominating some prices in L^0 and in L^1, but not being dominated by any point. Because the total length of segments of L^0 and L^1 that (p,e^0+x) dominates is constant for any such x, the concavity of G implies that it is more profitable to dominated L^0 alone. That is, this deviation is strictly less profitable than the deviation to (p,e^0+p-l), which is covered in case *(i)* as well.

Proposition 6

Consider a symmetric Nash equilibrium strategy μ that satisfies independence and constant comparability. The feature that the induced marginal distribution over q^k has no atoms and no holes carries over to the present setting. From now on, we adhere to the (p,e) representation of pure strategies. The proof proceeds stepwise.

Step 1: The marginal distribution over p is atomless.

Proof: Assume the contrary – i.e., that some price p is realized with positive probability. Then, with positive probability $p_1 = p_2 = p$. In this case, (p_1, e_1) and (p_2, e_2) necessarily do not dominate one another. Thus, conditional on $p_1 = p_2 = p$, the probability of domination is zero. By constant comparability, the probability of domination must be zero in equilibrium, in contradiction to Proposition 3.

The following two steps state properties that hold for *almost all* pairs of realizations of a symmetric equilibrium strategy. For expositional convenience, we state and prove the claims with slight imprecision, as if they hold for *all* realizations.

Step 2: If (p', e') is dominated by (p, e), then (p'', e') is dominated by (p, e) for every $p'' \in (p, p')$.

Proof: Let $p' > p'' > p$ be three prices in the support of the marginal distribution over p. By definition, if (p'', e') is dominated by (p, e), then (p', e') is dominated by (p, e) as well. Now, calculate the probability of domination conditional on $(p_1, p_2) = (p', p)$, by integrating over all possible values of e_1, e_2, and do the same for $(p_1, p_2) = (p'', p)$. By independence, e_1 and e_2 are *i.i.d.* Therefore, if (contrary to the claim) there is positive probability that (p', e') is dominated by (p, e) yet (p'', e') is not dominated by (p, e), we will get a violation of constant comparability, because the domination probability conditional on (p', p) will be strictly higher than the domination probability conditional on (p'', p).

Step 3: For every (p, e) and (p', e') in the support of μ with $e \neq e'$, $|e' - e| \geq |p' - p|$.

Proof: Assume the contrary, i.e., $|e - e'| < h - l$ for e, e' in the support of the marginal obfuscation distribution (where h and l are as defined in the proof of Proposition 5). By Step 1, we can find a price $p \in (l, h)$ in the support of the marginal distribution over p, such that $p - l < |e - e'|$. This means that (h,e) will be dominated by (l,e') and yet (p,e) will not be dominated by $(l.e')$, contradicting Step 2.

Step 4: The marginal equilibrium distribution over e is uniform with support $\{e^0,...,e^{n-1}\}$, where $e^{k+1}-e^k = h-l$ for every $k=0,...,n-2$, and $h+e^{n-1} = h-e^0 = 1$.

Proof: Step 3 immediately implies that the gap between two adjacent realizations $e < e'$ cannot be less than $h-l$. Assume the gap is strictly greater than $h-l$. Then, a competitor can profitably deviate from (h,e) to $(h,e+\delta)$, where $\delta > 0$ is arbitrarily small. The reason is that since the distribution over p is atomless, it assigns positive probability to prices arbitrarily close to h. Thus, by switching to $(h,e+\delta)$, the competitor reduces the probability of being dominated by strategies (p,e) for $p < h$, without affecting the probability of being dominated by strategies (p,e''), $e'' \neq e$. Since the marginal distributions over q^k have no holes, $h+e^{n-1} = h+e^0 = 1$. Finally, the reason that the distribution is uniform is as follows. In equilibrium, competitors are indifferent among all (h,e^k). By construction, the payoff from (h,e^k) is $h\cdot\frac{1-\lambda}{2}\cdot(1-\Pr(e^k))$, because (h,e^k) is dominated by (p,e) if and only if $p<h$ and $e=e^k$.

Step 5: $h = 1-\frac{n-1}{2}(h-l)$, $l = 1-\frac{n+1}{2}(h-l)$.

Proof: Recall that $h+e^{n-1} = h-e^0 = 1$. Therefore, $e^0 = -e^{n-1}$. Since values of e are evenly spaced by intervals of length $h-l$, it follows that the distribution of e is symmetric around zero, such that $e^{n-1} = \frac{n-1}{2}(h-l)$, and the result follows.

To complete the proof, we add the equation that the profits at l and h coincide:

$$h\cdot\frac{1-\lambda}{2}\cdot(1-\frac{1}{n}) = l\cdot\left[\frac{1-\lambda}{2}+(1-\frac{1-\lambda}{2})\cdot\frac{1}{n}\right]$$

This equation, coupled with Step 5, gives us the solutions to h and l, as well as the equilibrium profit, as a well-defined function of n. We can retrieve the marginal distribution G over p from the following equation:

$$\frac{\sigma(2\sigma n+2)}{2\sigma n+n+1}=p\cdot\left[\frac{1-\lambda}{2}\cdot\left(1-\frac{1}{n}+\frac{1}{n}(1-G(p))\right)+(1-\frac{1-\lambda}{2})\cdot\frac{1}{n}\cdot(1-G(p)\right]$$

Since this equation holds for every p in the support of G, the support of G cannot have holes inside $[l,h]$, for otherwise there would be an atom, contradicting Step 1.

Thus, any symmetric equilibrium strategy takes the form $s^*(n,d)$ where $d=h-l$. As for the bounds on n, it is easy to verify that if $n>1+\frac{3}{\lambda}$ then a deviation to (p,e) where $p=h+\frac{d}{2}$ and $e=1-p$ is profitable and if $n<1+\frac{1}{\lambda}$ then a deviation to (p,e) where $p=l+\frac{d}{2}$ and $e=e^0+\frac{d}{2}$ is profitable.

Proposition 7

First, observe that each firm can secure a strictly positive profit - e.g., by mixing uniformly over $\{(q^1,q^2)|q^1+q^2\leq 2\}$. Therefore, firms must earn strictly positive profits in symmetric Nash equilibrium. Now, if a firm offers a price-quality price (p,q), it will dominate the outside option only if $p\leq 0$, in which case it would earn non-positive profits. It follows that in equilibrium, firms will never offer alternatives that dominate the outside option. As a result, a firm will never offer a dominant option. Therefore, the consumer chooses a non-default firm if and only if it outperforms all other alternatives along the dimension he sampled. We have established that no firm will ever outperform the outside option along the price dimension. It follows that $p=2$ for every (p,q) in the support of the firms' equilibrium strategy - if (p,q) is in the support for some $p<2$, the firm can deviate to $(2,q)$ and get the same clientele -size.

We have thus concluded that in equilibrium, firms play $p=2$ and randomize over q according to some *cdf* F. By standard arguments in the tradition of Varian (1980), the support of F must be an interval $[0,\bar{q}]$. When a firm plays $(2,0)$, it is chosen only when the consumer was initially assigned to it and exhibits default bias. It follows that firms' equilibrium payoff is $2 \cdot \frac{1}{2}\lambda$. When a firm plays $(2, q)$, $q \in [0, \bar{q}]$, its payoff is

$$(2 - q) \left((1 - \lambda) \cdot \frac{1}{2} F(q) + \lambda \cdot \frac{1}{2} \right) = \lambda$$

which immediately gives the solution.