Extracting Interaction-Created Surplus

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   This paper demonstrates that the economic surplus which agents produce in bilateral interactions is extractable by an outside party having sufficient initial resources. The third party achieves this outcome using a class of "exclusive-interaction" contracts. A basic extractability result is shown to be robust to several extensions: competition among outside parties, multiplicity of interacting agents, and a dynamic extension with repeated opportunities to interact. Finally, some connections with the economic intermediation literature are drawn. *Journal of Economic Literature* Classification Number: C79   © 2000 Academic Press

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1. INTRODUCTION

   This paper demonstrates that the surplus created in bilateral interactions is extractable by outside parties. I introduce a family of contracts which a third party can offer to the interacting agents, prior to their interaction. When both agents accept these contracts they interact, but retain a substantially reduced share of the interaction's surplus. Nevertheless, it is shown through a basic example that the agents might accept such a contract in equilibrium. In other words, two selfish agents might voluntarily surrender the entire surplus that they jointly produce to outside

parties—"extractors," as I refer to such parties. I argue that such surplus extracting contracts are plausible on two accounts. First, they incorporate familiar features of real-life contracts. Second, both firms and intermediaries are natural institutions that can play the role of an extractor. Therefore, the question I end up exploring is why such contracts do not appear explicitly in reality. To address this question, I consider three extensions of the basic example, in order to examine its robustness. It turns out that the extractability result survives the extensions.

The Basic Example

Two agents (referred to as agents 1 and 2) are about to form some economic interaction (bilateral trade, joint production, etc.), which yields a fixed surplus of $2. This surplus is split equally between them. The exact specification of the interaction is immaterial for the presentation. Neither agent can produce any surplus on her own. Now consider a third party, agent 3, who has an initial endowment slightly larger than 1. Prior to the interaction between agents 1 and 2, agent 3 offers them a contract, denoted by $M1$. What follows is the version offered to agent i ($i = 1, 2$):

(1) If agent j also accepts the contract, agent i commits to interact with her. Agent 3 owns the interaction's surplus and pays w to agent i.

(2) If agent j does not accept the contract, agent i commits not to interact with her. In return, agent i receives x from agent 3.

Thus, the outcome that the contract specifies for agent i when she accepts the contract depends on whether agent j accepts an identical contract. The payment in clause (1) is referred to as the "*interaction payment*," and the payment in clause (2) is referred to as the "*no-interaction payment*." Assume that agent 3 can commit to this contract and enforce it. If both agents 1 and 2 reject $M1$, they interact and get $1 each (in what follows, payoffs are always stated in monetary terms; hence, the dollar signs are usually omitted). The strategic situation following agent 3's offer can therefore be modeled as a strategic game, represented by a 2×2 matrix: See Fig. 1.

Now, let $x > 1$, and keep x smaller than agent 3's initial endowment, in order to ensure that $M1$ is feasible. Let $w \in (0, 1)$ be arbitrarily small. Hence, "accept" is a dominant strategy for agents 1 and 2. Their payoff is w, whereas agent 3's payoff is $2 - 2w$. It follows that in (dominant-strategy) equilibrium, *the agents accept the third party's contract, and surrender almost the entire interaction-created surplus to her*. This is the paper's basic extractability result.

$M1$ is essentially an *exclusive-interaction* contract. When an agent accepts the contract, she commits to interact with the other agent if and only

	Reject	Accept
Reject	*1 , 1*	*0 , x*
Accept	*x , 0*	*w , w*

FIGURE 1

if the latter also accepts an identical contract. Furthermore, $M1$ provides each agent with incentives to avoid interaction with the other agent if the latter does not accept the contract. Exclusive-interaction contracts are abundant in reality. Economic intermediaries often employ them (e.g., real-estate brokers, retail stores, etc.); and the relationship between a firm and its workers is essentially based on exclusive interaction (with the firm exercising ownership rights over the surplus that its workers create). Both institutions mediate and contract on interactions which, ex ante, could be carried out directly. Therefore, they can naturally serve the role of an extractor. However, the exclusive-interaction contracts employed by these institutions do not seem to resemble the basic example. They lack an explicit distinction between "interaction" and "no-interaction" payments, and they do not normally include conditions such as clause (2) in $M1$. For example, while a typical real-estate brokerage contract stipulates that the buyer cannot trade *directly* with a potential seller that the broker brings, it would not explicitly provide the buyer with incentives to avoid trading with a seller whom the broker *did not bring*. The difference is significant, since the extractability result is driven by this very distinction.

There are few real-life contracts which seem to bear some resemblance to $M1$. For instance, underwriters help firms launch stock issues by committing to buy whatever residual amount of shares the public did not purchase. The underwriter exclusively mediates the transaction and insures the firm against an unsuccessful launch, and this insurance could be interpreted as a "no-interaction" payment. Such examples are rare, though, and their resemblance to $M1$ is admittedly superficial (in the underwriting example, no contract is signed between the underwriter and the potential buyers of the stocks, whereas in the basic example, the fact that a contract is offered to both agents is crucial to the extractability result). Why, then, do we not observe such contracts in reality? The subsequent sections consider some theoretical attempts to answer this question. In each section I discuss a different important element which is missing from the basic example, and extend the example to incorporate the new feature within a simple model. The extractability result is shown to be robust to the

extensions; none can therefore provide a successful theoretical counterargument to the original extractability claim.

Repeated opportunities to interact. The basic example displays a static, one-shot game. In the dynamic extension, I assume that the extractor's contracts are of a one-period term. This feature raises the possibility that the agents might prefer to postpone their interaction until the contracts expire. The extractor would then be able to offer them new exclusive-interaction contracts, and so forth. I allow the extractor to offer such contracts repeatedly, until the agents finally coordinate their actions and interact. It is shown that the extractor cannot produce an extractability result in this model by offering $M1$ repeatedly. However, if she can commit to a history-dependent payment strategy, a full-extraction outcome can be achieved.

Two-sided markets. The case of multiple interacting agents is studied using a two-sided market model. In this model, if the extractor fails to attract the entire population on one side of the market, she cannot persuade even a single agent on the other side to accept a surplus-extracting contract. In other words, the agents have a potentially richer variety of outside options than in the basic example, when facing the extractor's contract. Thus, it would seem that in order to produce an extractability result, the extractor might need a high (i.e., proportional to the population size) initial endowment. Nonetheless, I show that even if the extractor's initial endowment is the same as in the basic two-agents example, she can devise a payment scheme such that the unique rationalizable strategy for the agents is to accept her contract. This scheme extracts the entire market surplus.

Competition among extractors. The basic example studies the possible intervention of a single extractor. To analyze the entry of more than one extractor, I study a simple competitive model, and show that it has a continuum of equilibria. In particular, there is an equilibrium, in which one extractor behaves exactly like the monopolistic extractor in the basic example, while the competitors effectively stay out of the market. The persistence of the extractability result is shown (subject to some technical caveats) to be independent of the restriction to the case of only two interacting agents, provided that the extractor's initial endowment is sufficiently high.

Note that formally, the basic example is nothing more than a variety of the Prisoner's Dilemma, devised for the two agents by the extractor via an exclusive-interaction contract. I am aware of at least one precedent in the literature for a similar situation (Section III in Aghion and Bolton (1987)). In their paper, an incumbent seller offers exclusive contracts to potential buyers for entry deterrence purposes. In a simplified version of their model, the buyers' decision problem after the incumbent's offer is equiva-

lent to the Prisoner's Dilemma, so that in equilibrium the buyers accept the incumbent's contract and pay the monopoly price. There are two main differences between Aghion and Bolton (1987) and the basic example in the present paper. First, the incumbent in the former case takes an integral part in the interaction with the buyers, whereas in the present paper, the entire point is that she contributes nothing to the agents' interaction. Second, the exclusive contracts that are used in the two papers are different—there is no equivalent in Aghion and Bolton (1987) to the crucial no-interaction payment that clause (2) in $M1$ defines. At any rate, the similarity between the two cases breaks down in the extensions of the basic example—they make no sense if we interpret agent 3 as a seller.

The remainder of the paper is structured as follows: Section 2 analyses the dynamic extension; Section 3 analyses a two-sided market model with multiple interacting agents; and Section 4 studies the competitive extension. Section 5 concludes with a discussion and a note on the relation of this paper to the economic intermediation literature. Proofs appear in the Appendix.

2. A MODEL WITH REPEATED OPPORTUNITIES TO INTERACT

The basic example of Section 1 describes a static, one-shot situation. In this section I introduce a temporal element into the example. The extractor's contracts are of one-period term. As in the basic example, whenever the extractor offers the agents exclusive-interaction contracts, they interact if and only if both accept or reject the contracts. Unlike the basic example, in this section the game does not end in case the agents fail to coordinate. Instead, the agents are allowed to re-attempt interacting after the contracts expire, and the extractor can therefore offer them a new exclusive-interaction contract. This situation is repeated, and the process stops only when the agents finally coordinate and interact.

I represent the depicted scenario by the following multiperiod game. Time is measured by $t = 0, 1, 2, \ldots$. At every period $t \geq 1$, each of agents 1 and 2 faces a one-period exclusive-interaction contract in the form of $M1$. As in the basic example, these contracts define interaction and no-interaction payments for each agent. The payments, however, can be agent-specific and history-dependent. At every period, each agent has two available actions: "reject" and "accept." If both agents accept, they interact and produce the \$2 interaction surplus; they transfer it to the extractor, receive the interaction payments, and the game ends. If both agents reject, they interact and split the \$2 interaction surplus equally between them, and the game is terminated. If one agent accepts and the other rejects,

then no interaction occurs at that period; the accepting agent receives a no-interaction payment from the extractor, and the game proceeds to the next period. Agent 3, the extractor, moves only once, at period $t = 0$. She commits to a payment strategy, i.e., a function that assigns interaction and no-interaction payments for every agent and every nonterminal history of the subgame (between agents 1 and 2) that follows the extractor's move. The assumption that the extractor chooses her payment strategy at $t = 0$ and has no subsequent moves is discussed at the end of this section.

More formally, let $G(f)$ be the subgame between agents 1 and 2, that starts at period $t = 1$, after the extractor chooses a payment strategy f at $t = 0$. Denote agent i's action ($i \in \{1, 2\}$) at period t by $a_i(t)$ ($a_i(t) \in \{R, A\}$), where "R" denotes rejection of the one-period contract and "A" denotes its acceptance. A nonterminal history in $G(f)$ is a finite sequence, $((a_1(1), a_2(1)), (a_1(2), a_2(2)), \ldots, (a_1(T), a_2(T)))$, such that $a_1(t) \neq a_2(t)$ for all $t = 1, 2, \ldots, T$. Let H be the set of nonterminal histories in $G(f)$ (H also includes the empty history, for the case of $T = 0$). A strategy for the extractor is a function $f : H \times \{1, 2\} \to \Re^2$ that assigns to every nonterminal history $h \in H$ and every agent $i \in \{1, 2\}$, interaction and no-interaction payments for the period which immediately follows h, denoted $w_i(h)$ and $x_i(h)$, respectively. I assume that all players (agents 1 and 2, as well as the extractor) maximize the discounted sum of their periodic payoffs, the discount factor common to all players being δ ($\delta < 1$). The extractor has some finite initial endowment.

Any payment strategy must satisfy a feasibility constraint, namely, that the maximal discounted sum of the no-interaction payments that the extractor might incur in any path of $G(f)$ cannot exceed her initial endowment. Suppose that the extractor repeatedly offers the same $M1$ of the basic example (with $x > 1$ and $w > 0$ arbitrarily small, so that in the one-shot game, A would dominate R and the interaction surplus would be virtually extracted). In this case, the extractor's initial endowment must exceed $1/(1 - \delta)$ for this scheme to be feasible. More important, this payment strategy gives rise to a large class of subgame perfect equilibria, in which the two agents deliberately and repeatedly delay their interaction. For instance, $((R, A), (A, R), (R, A), (A, R), \ldots)$ is a subgame perfect equilibrium path if δ is sufficiently large. The high no-interaction payments induce agents 1 and 2 to delay their interaction, and drive the extractor to bankruptcy.

This example raises the following question: Is there a payment strategy for the extractor, which induces agents (via subgame perfection) to play A at an early stage of the game while retaining the extractability result? Consider the following payment strategy, denoted by $M2$. Let $\varepsilon > 0$ be arbitrarily small. For every nonterminal history $h = ((a_1(1), a_2(1)),$

$(a_1(2), a_2(2)), \ldots, (a_1(T), a_2(T)))$ (where $T = 0, 1, 2, \ldots$):[1]

$$w_i(h) = \begin{cases} 1 & \text{if } T > 0 \quad \text{and} \quad a_i(T) = A \\ \varepsilon & \text{if } T = 0 \quad \text{or} \quad \left[T > 0 \text{ and } a_i(T) = R \right] \end{cases}$$

$$x_i(h) = X \quad \text{where } 1 - \delta < X < 2 - 2\delta.$$

This payment scheme has a simple interpretation; at every period, it distinguishes between "loyal" and "disloyal" agents. An agent is considered loyal at period t if she played A at period $t - 1$. Otherwise, she is considered disloyal (both agents are considered disloyal at $t = 1$, and an agent can be considered loyal only from period $t = 2$ onward). A loyal agent receives her full share of the interaction's surplus in case of interaction, whereas the disloyal agent's share is virtually extracted. X, the no-interaction payment, is history-independent, so that loyal and disloyal agents alike receive the same no-interaction payment. Note that the execution of $M2$ requires only one-period recall. An additional important feature of $M2$ is that the maximal discounted sum of the no-interaction payments that the extractor might incur in any path of $G(M2)$ is $X/(1 - \delta)$. Therefore, the initial endowment required by $M2$ can be the same (in discounted terms) as in the basic, one-shot example. I now show that $G(M2)$ has a unique subgame perfect equilibrium with full surplus extraction.

PROPOSITION 1. $G(M2)$ *has a unique subgame perfect equilibrium, in which the strategy of both agents* 1 *and* 2 *is to play always* A. *In particular, they play* A *at* $t = 1$ *and the extractor's equilibrium payoff is* $2 - 2\varepsilon$.

To understand how $M2$ works, recall that the extractor's problem was to offer sufficiently high no-interaction payments to attract the agents, and to ensure at the same time that the agents do not exploit these payments by deliberately delaying their interaction. The structure of interaction and no-interaction payments in $M2$—higher interaction payments to loyal agents, as well as no-interaction payments that are sufficiently high to induce each agent to play A when her opponent is expected to play R (but not so high as to make deliberate miscoordination profitable)—ensures that once an agent plays A, she sticks to A, regardless of her opponent's strategy. The opponent has no alternative but to follow her footsteps and play A as well. Both agents therefore play A as soon as they can. The result is independent of the measure of the agents' time preferences, so long as $\delta < 1$.

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[1] Recall that the payments $w_i(h)$ and $x_i(h)$ apply to period $T + 1$.

The equilibrium outcome does not rely on any off-equilibrium ineffi-ciencies, since the agents choose to interact after every nonterminal history. Hence, $M2$ is *renegotiation-proof*. In other words, if we maintained the assumption that the extractor cannot withdraw unilaterally from $M2$ after $t = 0$, but allowed the players to renegotiate the extractor's contracts at any stage of the game, the extractability result would not be affected. In particular, according to the renegotiation-proofness criterion, any attempt by the extractor to back up from $M2$ would be blocked by one of the agents.

Suppose that we gave up the assumption that the extractor moves only at $t = 0$. Instead, let a two-stage game—in which the extractor chooses a one-period contract at the first stage and the agents simultaneously decide whether to accept it at the second stage—be repeatedly played until the first time the agents choose the same action, whereupon they interact and the game ends. Here, too, we are interested in the set of subgame perfect equilibria, except that now the perfection of the extractor's strategy is also scrutinized. Recall that $M2$ fully extracts the interaction surplus only at $t = 1$. For all $t > 1$, it extracts only the share of the agent that is considered disloyal at period t. Suppose now that at some $t > 1$, the extractor deviates from the proper continuation of $M2$, and instead starts $M2$ all over again. By Proposition 1, the agents will still play A at period t, only now the entire interaction surplus will be extracted. Hence, $M2$ is not a credible strategy for the extractor, and cannot be part of subgame perfect equilibrium in this extended model. The assumption that the extractor chooses her payment strategy at $t = 0$ and does not get to move at any $t > 0$ is therefore crucial for the extractability result of this section.

3. A TWO-SIDED MARKET MODEL

In the basic example there are two interacting agents, and the extractor needs to be endowed with slightly more than \$1 in order to carry out $M1$. The present section investigates the effect of increasing the number of agents on the extractor's problem. To model the case of one extractor and multiple interacting agents, consider a two-sided market, which consists of N identical risk-neutral agents on each side (the case of unequal numbers of agents on the two sides is discussed later). N can be arbitrarily large. Agents on one side of the market interact only with agents on the other side. In order to facilitate the exposition, I often refer to agents on one side of the market as "men" and to agents on the other side as "women." As in the basic example, there is no market inefficiency to begin with. Every man and woman interacts and receives a payoff of 1.

Enter the extractor, endowed with some amount of initial resources. As in the basic example, the extractor offers exclusive-interaction contracts to the agents, prior to their intended interactions. This induces a simultaneous-move game among the agents, who choose between R (rejecting the extractor's contract) and A (accepting the extractor's contract). Denote the number of men (women) who choose action $i \in \{R, A\}$ by p_i (q_i). I refer to agents (men, women) who choose action "i" as i-players (i-men, i-women). R-players on one side are randomly matched with R-players on the other side. The expected payoff of R-men and R-women is thus Min$(1, q_R/p_R)$ and Min$(1, p_R/q_R)$, respectively.

The payoff of A-players is determined by the extractor's contracts. As in the two-agents basic example, the contracts specify interaction and no-interaction payments. However, there is one natural difference with respect to $M1$ of the basic example. An agent who accepts $M1$ is obliged to interact (avoid interaction) with a specific opponent if the opponent accepts (rejects) $M1$. In comparison, contracts in the market model can be more general in the way they determine whether an A-player is obliged to interact or to avoid interaction (e.g., contracts can require that the interaction is with an agent that belongs to some subset of the population on the other side of the market; they can involve random matching; etc.). Thus, the extractor's problem in the market model is to choose a function that specifies (possibly probabilistically) for every play profile of the agents in the two-sided market, which of the A-players interact, as well as interaction payments for the A-players who interact and no-interaction payments for those who do not. This function is subjected to two feasibility constraints. First, the number of interacting A-players on either side cannot exceed Min(p_A, q_A). Second, the net loss that the extractor may incur in any play profile of the agents cannot exceed her initial endowment.

Suppose that the extractor chooses the following scheme, which is a natural extension of $M1$. Given p_A and q_A, every A-man interacts with probability Min$(1, q_A/p_A)$ and every A-woman interacts with probability Min$(1, p_A/q_A)$. Every A-player receives \$$w$ if he/she interacts and \$$x$ if he/she does not (w and x are the same as in the original $M1$ of the basic example). First, note that the extractor's initial endowment cannot be less than $xN > N$ in order to carry out this scheme. Second, accepting the extractor's contract is no longer a dominant strategy for the agents in the game induced by this scheme. To see that, suppose that half the men and half the women accept the contract. Thus, every A-player interacts and receives w from the extractor. The A-player can profitably deviate to R, increasing his/her expected payoff from w (which is arbitrarily small) to $N/(N+2)$. In general, in order to persuade any individual agent on one side of the market to accept a surplus extracting contract, the *entire* population on the other side must be persuaded to accept the extractor's

contracts as well. Otherwise, an individual agent would prefer to reject the contract, since she will interact with positive probability with "rejecters" on the other side of the market.

A straightforward modification of the extended $M1$ can make A a dominant strategy for all agents in the induced game. In this version, the interaction payment of an A-player equals the infinitesimal w only when all agents on the other side play A; otherwise, the agent's interaction and no-interaction payments are $x > 1$. Like the above extension of $M1$, this version requires that the extractor's initial endowment be larger than N. The question is whether the extractor can achieve full extraction with a small endowment. It is shown below that there exists a payment scheme that requires (independently of N) an initial endowment slightly larger than 1 (as in the basic example), such that A is the unique rationalizable strategy for the agents in the induced game. Let $M3$ be the following deterministic scheme employed by the extractor:

1. Set all agents on each side according to some arbitrary predetermined order, such that it is possible to speak of the ith man or jth woman $(i, j = 1, \ldots, N)$. This ordering is common knowledge among the players.

2. If $p_A > q_A$, let all A-women interact. Set the A-men according to their predetermined order. Let the top q_A A-men according to the list interact, and do not let the bottom $p_A - q_A$ A-men interact. Pay $1 + \varepsilon$ ($\varepsilon > 0$ is arbitrarily small) to the $(q_A + 1)$th A-man according to the list. Pay nothing to the remaining unmatched men.

3. If $p_A < q_A$, let all A-men interact. Set the A-women according to their predetermined order. Let the top p_A A-women according to the list interact, and do not let the bottom $q_A - p_A$ A-women interact. Pay $1 + \varepsilon$ to the $(p_A + 1)$th A-woman according to the list. Pay nothing to the remaining unmatched women.

4. If $p_A = q_A$, let all A-women and A-men interact.

5. When an A-player interacts with an A-player on the other side of the market, the extractor owns the interaction's surplus; the agent receives ε if $p_A = q_A = N$, and $1 + \varepsilon$ otherwise.

It can be seen that the initial endowment required by $M3$ is slightly bigger than 1. At most, the extractor makes a no-interaction payment of $1 + \varepsilon$ to one agent, and a net payment of ε to a finite number of agents. Before stating this section's extractability result, let us look at this mechanism through an example. Let $N = 5$, and suppose that four men, placed 1st, 3rd, 4th, and 5th on the predetermined order—as well as two women—play A. Then, the two A-women interact and receive $1 + \varepsilon$ from the extractor; the 1st and 3rd men also interact and receive $1 + \varepsilon$; the 4th man does not interact, and he receives $1 + \varepsilon$ from the extractor. The 5th man does not

interact, and he receives nothing. The extractor's payoff in this configura-
tion is therefore $2 \cdot 2 - 4 \cdot (1 + \varepsilon) - (1 + \varepsilon) = -1 - 5\varepsilon$.

PROPOSITION 2.   *The unique rationalizable strategy in the game induced by*
$M3$ *is* A.

Proposition 2 shows that it is possible to extract the aggregate interac-
tion surplus of a large market with a small initial endowment, simply by
ordering the agents in a "line" for the extractor's "matchmaking services."
There is an unrealistic aspect of $M3$, namely, that its interaction payment
is infinitesimal only when the entire market chooses A, and very large in
all other play profiles. This drawback can be overcome by constructing a
more elaborate payment scheme, in which the extractor makes relatively
small interaction payments in many more play profiles. Such an elabora-
tion would have unnecessarily complicated the exposition of the result.

$M3$ is devised for a symmetric two-sided market, where there are equal
numbers of men and women. For the asymmetric case, where there are N_1
men and N_2 women ($N_1 > N_2$, without loss of generality), a small change
in the details of $M3$ suffices to generate the extractability result. There is
no change in the structure of interaction payments in the modified $M3$.
Also, the no-interaction payment of the first unmatched A-man according
to the list is $1 + \varepsilon$, as in $M3$. The only change with respect to $M3$ is that
the remaining unmatched A-men's no-interaction payment is ε (as op-
posed to 0 in M3). This small change assures (without increasing the
extractor's necessary initial endowment) that A is the unique rational-
izable action for the last $N_1 - N_2$ men on the predetermined order,
even though in equilibrium they have no chance to get matched when
playing A.

4. COMPETITION BETWEEN EXTRACTORS

The basic example assumes that only one extractor enters the market. In
this section I analyze the case of two competing extractors, wherein each
extractor tries to induce both agents to accept her contract. The following
analysis aims to investigate how competition between extractors affects
their rents, and whether it increases the agents' equilibrium payoffs. This
very simple analysis generalizes to an arbitrary number of competing
extractors.

The situation is modeled as a two-stage game. At the first stage, the two
extractors simultaneously offer exclusive-interaction contracts in the form
of $M1$ (the payments, of course, can be different) to agents 1 and 2. At the
second stage, having observed the extractors' contracts, agents 1 and 2
choose from among three options: rejecting both extractors' contracts (R),

	R	A	B
R	$1,1$	$0, x_A$	$0, x_B$
A	$x_A, 0$	w_A, w_A	x_A, x_B
B	$x_B, 0$	x_B, x_A	w_B, w_B

FIGURE 2

accepting the first extractor's contract (A) and accepting the second extractor's contract (B). Extractor i's contract $(i \in \{A, B\})$ is a pair, (x_i, w_i), where x_i is the payment to an agent when she alone accepts the contract (i.e., the no-interaction payment), and w_i is the payment to an agent when both agents accept it (i.e., the interaction payment).[2] If both agents accept extractor i's contract, they interact, transfer the interaction surplus to extractor i, and receive w_i. If both agents reject both extractors' contracts, they interact and receive 1 each. If one agent receives extractor i's contract and the other agent rejects it, then no interaction occurs and the former agent receives x_i. Each extractor has an initial endowment of $1 + \varepsilon$ ($\varepsilon > 0$ is arbitrarily small). $x_i \in [0, 1 + \varepsilon]$ and $w_i \in [0, 1]$. The play between the two agents at the second stage can therefore be represented by a 3×3 matrix: See Fig. 2.

I now explore the set of pure-strategy subgame perfect equilibria in the two-stage game:

PROPOSITION 3.   *Let $i, j \in \{A, B\}$, $i \neq j$. Then, all pure-strategy subgame perfect equilibria of the two-stage game satisfy: $x_i \geq 1$, $w_i = x_j$, agents 1 and 2 play i, and w_i can be anything in $[0, 1]$.*

The significance of this result is that competition between extractors cannot rule out the quasi-monopolistic solution, in which the interacting agents' surplus is fully extracted by one extractor. Indeed, a continuum of interaction payments is possible in equilibrium. Some degree of surplus extraction occurs in every equilibrium. In all of these equilibria the payoff of the low-x extractor is null. What drives this result is that the extractors compete in their no-interaction payments as well as in their interaction

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[2] The results of this section do not change if I allow the extractors' contracts to distinguish between agents who accept the rival extractor's contracts and agents who reject all contracts. Also, I do not allow the agents to accept two contracts at the same time, but this does not affect the results, since we assume that they can interact only once and that the extractors observe their interaction.

payments—they can attract agents to accept stringent contracts by offering a high no-interaction payment, which is not actually made in equilibrium. As far as the degree of surplus extraction is concerned, Proposition 3 is weaker than the results of the preceding sections, because of the multiplicity of equilibria it predicts. However, it should be stressed that the multiplicity of equilibria in this simple model cannot explain why we fail to observe contracts with the *structure* of $M1$ in reality, since contracts with positive no-interaction payments are offered in *every* equilibrium. Thus, the competitive model cannot account for a world without extractors altogether.

Note that small changes in the structure of the model may isolate the monopolistic outcome. Consider the following two (independent) extensions of the basic competitive model. First, if the extractors' choice of contracts were sequential, there would be a unique subgame perfect equilibrium, in which the first mover offers a fully extracting contract with $w = 0$ and $x \geq 1$, the second mover offers $x = 0$, and the agents accept the former extractor's contract. Second, suppose that we assumed (in the simultaneous-move model) that offering every contract other than $(0, 0)$ is costly. Then, in equilibrium, one extractor offers a fully extracting contract with $w = 0$ and $x \geq 1$, whereas her opponent ''stays out of the market'' and chooses $x = w = 0$ (this result follows from the observation that in the original simultaneous-move model, the low-x extractor's profits are null).

The question arises, whether the persistence of the quasi-monopolistic outcome in equilibrium relies on the restriction of the model to the case of only two interacting agents. Let us consider a model with competition between extractors over a two-sided market with N agents on each side. For simplicity, assume that N is even. A two-stage game is played. At the first stage, two extractors simultaneously offer exclusive-interaction contracts to the agents, and at the second stage, every agent chooses from among three options: rejecting both extractors' contracts (R), accepting the first extractor's contract (A) and accepting the second extractor's contract (B). A payment scheme for extractor i $(i \in \{A, B\})$ is a function that determines for every play profile in the second stage, which of the i-players interact, as well as interaction and no-interaction payments for each i-player (as in Section 3, the scheme can be probabilistic). Any scheme is subjected to the same feasibility constraints that applied in Section 3.

Let extractor A's initial endowment be $N(1 + \varepsilon)/2$ (where $\varepsilon > 0$ is arbitrarily small), and consider the payment scheme $M3^*$, which is identical to $M3$, except for the following detail. Let $k^* = \text{Min}(|p - q|, N/2)$, where p (q) is the number of A-men (women). Then, the first k^* unmatched agents, according to the predetermined order specified by

$M3^*$, receive a no-interaction payment of $1 + \varepsilon$ (whereas by $M3$, only the *first* unmatched agent according to the predetermined order is paid $1 + \varepsilon$). The remaining unmatched A-players receive nothing.

PROPOSITION 4. *In any play profile, in which extractor A's strategy is fixed to be $M3^*$ and all other players choose a best response (both to extractor A's strategy and to each other's), it must be that all agents play A at the second stage and extractor B's no-interaction payments do not exceed ε. Extractor A's payoff is $2N(1 - \varepsilon)$.*

This proposition illustrates how a sufficiently large initial endowment for one extractor can maintain the quasi-monopolistic outcome in a competitive environment with many interacting agents. $M3^*$ ensures that the competing extractor effectively stays out of the market. Note that the play profiles referred to in Proposition 3 are not equilibria, due to standard considerations concerning the openness of the set of all $M3^*$ schemes.[3] If, however, payments are restricted to multiples of ε, every such play profile constitutes a subgame perfect equilibrium. As in the two-agents model, if the extractors' moves were sequential, the first mover could guarantee full extraction of the market surplus in the subgame following her move.

To see why the choice of $M3$ (rather than $M3^*$) by one extractor does not guarantee that the opponent's best response will be to stay out of the market, let us look at an example. Let $N > 4$. Extractor A chooses $M3$, whereas extractor B employs the following simple payment scheme. B-players on one side of the market are randomly matched with B-players on the other side. The women's interaction and no-interaction payments are a constant $1 + \varepsilon$, and the men's interaction and no-interaction payments are a constant ε. Denote the number of i-men (women) by p_i (q_i). The configuration $p_R = q_R = 0$; $q_A = q_B = N/2$; $p_A = N/2 + 1$; $p_B = N/2 - 1$, can be sustained by a second-stage equilibrium, if the B-men are all at the bottom of extractor A's list. In that case, although the B-men's payoff is ε, they have no incentive to play A, since they will not interact and receive a zero no-interaction payment. In this second-stage equilibrium, extractor A's payoff is $-1 - \varepsilon(N + 1)$, and extractor B's payoff is $N(1/2 - \varepsilon) - 2 + \varepsilon$, which is positive for $N > 4$. Therefore, $M3$ cannot guarantee full extraction in equilibrium. The question whether full extraction can emerge in equilibrium with payment schemes which require an initial endowment that is not proportional to the population size is left for future research.

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[3]$M3^*$ relies on ε being strictly positive, and every $\varepsilon > 0$ defines a different scheme in the form of $M3^*$. Thus, the set of all $M3^*$ schemes is open.

5. CONCLUDING REMARKS

In the previous sections I demonstrated that bilateral interactions were vulnerable to the extraction of surplus by outside parties. The contracts $M2$ and $M3$ introduce further familiar contractual elements, in addition to those discussed in the introduction. In $M2$, it is the benefits to ''loyal customers''; in $M3$, it is the ordering of agents in a line, so that agents in a better position on the line have better chances to get matched and receive positive payments. These elements can all be detected in real-life mechanisms. The puzzle raised in this paper is that these specific combinations of contractual features are not observed in reality.

The Agents' Inability to Write Exclusive-Interaction Contracts

I have saved one objection to this section. Throughout the paper, I have permitted only agent 3 to do the contracting. However, one might argue that the interacting agents could write, in advance, a bilateral exclusive-interaction contract with punitive clauses, which force them to reject the extractor's contract, and even prevent them from interacting with any other agent.[4] For sufficiently large penalties inflicted on deviant agents, rejecting the extractor's contract when it is offered would be the unique dominant strategy for both agents. While this is the most successful theoretical objection to the extractability result that I have come across so far, I do not think it answers the puzzle, for the following reasons.

1.   The agents may be unable to communicate before their interaction takes place because of search problems. In particular, they may have to bear search costs and ''show up'' in a market in order to ''find'' each other. Thus, it would be impossible for them to reach a bilateral exclusive-interaction agreement beforehand. I have assumed away search problems in this paper. However, the results presented here do not depend on this assumption.

2.   I think of the original interaction between agents 1 and 2 as a ''big'' one (e.g., a real-estate transaction, a long-term employment relationship, marriage, etc.). Therefore, it is reasonable to expect that the extractor is able to contract on the interaction's occurrence. In contrast, the interaction of agents 1 and 2 with the extractor is less observable and less verifiable. Thus, agents 1 and 2 may sign in advance a contract that prohibits them to interact with any third party, but this does not prevent agent 1, say, from secretly signing an exclusive-interaction contract with

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[4] I thank Elhanan Ben-Porath for this argument.

the extractor.[5] Agent 2 may then face agent 1's refusal to interact with her, without knowing (and much less being able to verify) the reason. In short, there may be a natural asymmetry in the players' ability to offer exclusive-interaction contracts.

3. For precontracting between agents 1 and 2 to take place, the two need to interact. However, this interaction can be equally blocked in advance by some third party, by the same arguments I have used in this paper. It can be argued that each agent can block the extractor's intervention *unilaterally* by writing a unilateral contract in which she commits to avoid interacting with third parties.[6] However, this seems like replacing one unrealistic contract with another. First, such a unilateral exclusivity contract is naturally hard to enforce. Second, it is strategically unstable (if one agent commits never to accept third parties' exclusive-interaction contracts, the other agent prefers not to make an equivalent commitment). Finally, it may be difficult to anticipate in advance the interactions in which intervention by third parties will be unwarranted by the agents.

The Relation to the Intermediation Literature

As mentioned in the Introduction, the extractor concept has natural institutional interpretations, and there is no need to invent new ones for this purpose. There are many economic institutions whose functioning depends upon the interaction of other agents, most notably, economic intermediaries. The literature on economic intermediaries typically assumes that they serve some "role" in the economy, in the sense that they propose solutions to some "friction" in the direct market interaction. Financial intermediaries offer risk-sharing in incomplete markets; employment agencies and real-estate brokers possess better matching technologies and information in two-sided markets; etc. However, in the basic example of this paper and its extensions, there is no friction to begin with. The intervention of the third party eventually reduces the agents' welfare yet leaves the total interaction surplus unchanged.

The closest form of intermediation to the extractor's activity, especially in relation to Section 3, is matchmaking. Matchmakers are distinguished from other types of intermediaries, such as experts or marketmakers, in that their main function is to match two or more agents, without participating in their actual interaction. Matchmakers are particularly abundant in real-estate, marriage, and labor markets. However, there are very few

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[5]One needs to distinguish between a contract that prohibits beforehand any future interaction with a third party and a contract which forces the agents to interact with each other at a specified future date. I consider the latter tantamount to an actual interaction between the agents.

[6]I thank Eddie Dekel for this idea.

papers that I know of which study matchmakers as such in a noncooperative framework. The exceptions are Yavas (1992) and Bloch and Ryder (1994). For models of other types of intermediaries, see Biglaiser (1993), Biglaiser and Friedman (1994), Gehrig (1993), Rubinstein and Wolinsky (1987). Models of matchmakers inherently include an element of strategic complementariness which causes a multiplicity of equilibria (e.g., Bloch and Ryder (1994)). The reason is that agents on the two sides of the market face a dilemma between interacting directly and interacting through the matchmaker. If an agent on one side of the market expects the majority of agents on the other side to interact directly, she also tends to avoid the matchmaker. If, on the other hand, she expects that most agents on the other side of the market interact exclusively through the matchmaker, she also tends to interact through the matchmaker. Proposition 2 implies that a monopolistic matchmaker can solve this problem and extract the entire market surplus, whereas Proposition 3 suggests that this result may be extended to the case of competition between matchmakers.

APPENDIX: PROOFS

Proof of Proposition 1.   The proof is presented stepwise. In the first step, I show that in equilibrium, agents 1 and 2 play A for all $t > 1$. In the second step, I show that they play A at $t = 1$.

Step 1.   Let $h = ((a_1(1), a_2(1)), \ldots, (a_1(t-1), a_2(t-1)))$ be a nonterminal history of length $t - 1$ ($t > 1$) in $G(M2)$. Assume that the equilibrium strategies do not prescribe $a_1(t) = a_2(t) = A$ immediately after h. I divide the class of subgame perfect equilibrium paths in the subgame $G(h)$ into four categories, and show that none can exist:

1.1. $a_1(t') \neq a_2(t')$ for all $t' \geq t$. Note that any unilateral deviation from such a path terminates the game. Note also that the sum of the agents' continuation payoffs at any period along this path is $X/(1-\delta)$. Since $X < 2(1-\delta)$, at least one agent—say, agent 1, without loss of generality—has a continuation payoff strictly below 1. Now, either of the following three alternatives holds. (i) $a_1(t) = A$. In this case, agent 1 can profitably deviate to R at period t, and increase her continuation payoff to 1. (ii) $a_1(t') = R$ for all $t' \geq t$. In this case, agent 1 can profitably deviate at period t to A, and increase her continuation payoff from 0 to at least ε. (iii) $a_1(t) = R$, and there exists $t^* > t$, such that $a_1(t^*) = A$ and $a_1(t^* - 1) = R$. In this case, if agent 1's continuation payoff at period t^* is strictly below 1, she can profitably deviate at period t^* to R, and increase her continuation payoff to 1. Conversely, if her continuation payoff at t^* is at least 1, then agent 2's continuation payoff at t^* must be strictly below 1.

Since $a_2(t^* - 1) = A$, it is profitable for agent 2 to deviate at period t^* to A—her interaction payment at t^* is 1. It follows that neither of the three alternatives is consistent with equilibrium.

1.2. $a_1(t^*) = a_2(t^*) = A$ for some $t^* > t$ (obviously, $a_1(t') \neq a_2(t')$ for all t', $t^* > t' \geq t$). Agent i, for whom $a_i(t^* - 1) = R$, can profitably deviate to A at period $t^* - 1$, which will increase her continuation payoff from $\delta\varepsilon$ to at least ε. Hence, such a path cannot be sustained in equilibrium.

1.3. There exist t^* and t'', $t^* > t'' \geq t$, such that $a_1(t^*) = a_2(t^*) = R$ and $a_1(t'') \neq a_1(t - 1)$ (again, $a_1(t') \neq a_2(t')$ for all t', $t^* > t' \geq t$). Therefore, there is a period t^{**}, $t^* > t^{**} \geq t$, and an agent i, such that $a_i(t^{**} - 1) = A$ and $a_i(t') = R$ for all t', $t^* \geq t' \geq t^{**}$. Agent i can thus profitably deviate from R to A at period t^{**}, increasing her continuation payoff from $\delta^{t^* - t^{**}}$ to 1. Hence, such a path cannot be sustained in equilibrium.

1.4. By steps 1.1–1.3, there remains one possible category of equilibrium paths in $G(h)$: $a_1(t^*) = a_2(t^*) = R$ for some $t^* \geq t$, and $a_i(t') = a_i(t - 1)$ for all $t^* > t' \geq t$, $i = 1, 2$ (again, $a_1(t') \neq a_2(t')$ for all t', $t^* > t' \geq t$). Assume, without loss of generality, that $a_1(t^* - 1) = A$. Now suppose that agent 1 deviates at period t^* from R to A. Then, by steps 1.1–1.3, the equilibrium path of the subgame which follows the deviation belongs to the present category—i.e., there exist some period $t^{**} > t^*$, such that $a_1(t') = A$ and $a_2(t') = R$ for all $t^{**} > t' > t^*$, and $a_1(t^{**}) = a_2(t^{**}) = R$. Agent 1's continuation payoff at period t^* in this case is $X + \delta X + \cdots + \delta^{t^{**} - t^* - 1}X + \delta^{t^{**} - t^*} \cdot 1$. Since $X > 1 - \delta$, this payoff is greater than 1. Thus, it is profitable for agent 1 to deviate from R to A at period t^*.

Therefore, no category of paths can be supported by subgame perfect equilibrium, so that any subgame perfect equilibrium of $G(h)$ requires that both agents play A at period t. it is easy to check that this is indeed an equilibrium.

Step 2. Without loss of generality, consider agent 1's strategic calculations at period $t = 1$:

2.1. Suppose that agent 1 conjectures that agent 2 plays A at $t = 1$. If she plays R, then by step 1, both agents will play A at $t = 2$, and her discounted payoff at $t = 1$ is $\delta\varepsilon$. If, however, she plays A, the game ends and she obtains ε. Therefore, her best response is A.

2.2. Suppose that agent 1 conjectures that agent 2 plays R at $t = 1$. Then, if she also plays R, the game ends and her payoff is 1. On the other hand, if she plays A, then, by Step 1, both agents will play A at $t = 2$, and

her payoff is $X + \delta \cdot 1$. Since $X > 1 - \delta$, this payoff is greater than 1. Therefore, her best response is A.

It follows that in subgame perfect equilibrium, both agents play A after every nonterminal history of $G(M2)$. In particular, they play A at period $t = 1$. By $M2$, each receives ε, whereas the extractor obtains $2 - 2\varepsilon$.

Proof of Proposition 2.   The proof is executed by iterative elimination of strictly dominated strategies. The induction is on the position of the agents on the predetermined lists. Without loss of generality, I review the men's side:

Step 1.   Consider the strategic calculation of the first man on the list:

(I) If $q_A = 0$, then, being the first on the men's list, if he plays A he receives a no-interaction payment of $1 + \varepsilon$, whereas if he plays R he receives 1.

(II) If $q_A = N$, then the first man interacts and receives at least ε if he plays A, while if he plays R, he receives zero.

(III) For all $1 < q_A < N$, if the first man plays A, he interacts and receives $1 + \varepsilon$, which is more than the maximum he can obtain if he plays R.

Therefore, R is a strictly dominated strategy for the first agent on each side.

Step 2.   Assume that R is eliminated by all first $k - 1$ agents on each side's list ($k = 1, 2, \ldots, N$), and check the kth man's strategic calculation:

(I) If $q_A = k - 1$, then if he plays A, he does not interact. He receives a no-interaction payment of $1 + \varepsilon$ (since all $k - 1$ men preceding him on the list play A, while he precedes the remaining A-men), which is more than the maximum he can obtain if he plays R.

(II) If $q_A = N$, then he interacts and receives at least ε if he plays A, while if he plays R, he obtains zero.

(III) For all $k - 1 < q_A < N$, if he plays A, he interacts and receives $1 + \varepsilon$, which is more than the maximum he obtains by playing R.

Therefore, R is a dominated strategy for the kth player on each side's list.

Proof of Proposition 3.   The play of agents 1 and 2 at the second stage is denoted by (i, j), where $i, j \in \{R, A, B\}$. First, suppose that $x_A, x_B > 1$ in equilibrium. Then, the second stage equilibria are either (A, B) or (B, A), and both extractors' payoff is negative. Therefore, it is profitable for any of the extractors to deviate to $x = 0$ at the first stage. It follows that in equilibrium, at least one extractor offers $x \leq 1$.

Now, suppose that $x_A, x_B < 1$ in equilibrium. Then (R, R) is a Nash equilibrium at the second stage. Furthermore, if $w_A \geq x_B$, then (A, A) is also a Nash equilibrium at the second stage; if $w_B \geq x_A$, then (B, B) is a Nash equilibrium at the second stage. Finally, if $w_A \leq x_B$ and $w_B \leq x_A$, then (A, B) and (B, A) are Nash equilibria in the second stage. In each of these equilibria, there is an extractor i, whose payoff is nonpositive, and who can deviate in the first stage to a contract satisfying $x_i > 1$ and $1 > w_i > x_j$ (where j is i's opponent). This deviation guarantees that i is a strictly dominant action for the agents at the second stage.

It remains that in equilibrium, it must be that either $x_A \geq 1$ and $x_B \leq 1$, or $x_B \geq 1$ and $x_A \leq 1$. In the former case, $w_A = x_B$ and the agents play A. In the latter case, $w_B = x_A$ and the agents play B. The existence of such equilibria can be easily verified for all $w_i \in [0, 1]$ ($i \in \{A, B\}$).

Proof of Proposition 4. Fix extractor A's strategy to be $M3^*$. Denote the number of men (women) who choose action $i \in \{R, A, B\}$ in the second stage equilibrium by p_i (q_i). $p_R + p_A + p_B = q_R + q_A + q_B = N$.

Step 1. If $p_B > 0$ and $q_B > 0$, then $p_B \neq q_B$.

Proof. Assume the contrary, i.e., $p_B = q_B > 0$. Suppose that $p_A \neq q_A$ —and, by implication, that $p_R \neq q_R$. Hence, there is an R-player whose expected payoff is below 1. If he/she deviates to A, then by $M3^*$, he/she interacts and receives $1 + \varepsilon$. Thus, it must be that $p_A = q_A$ (and $p_R = q_R$). Now, the B-players produce a total surplus of $2p_B$ at most. If the payoff of all B-players is above 1, then extractor B makes negative profits, which cannot be the case in equilibrium. Therefore, there must be at least one B-player whose payoff does not exceed 1, whatever the payment scheme of extractor B. This agent would prefer to deviate to A, since according to $M3^*$, he/she would receive a no-interaction payment of $1 + \varepsilon$. Thus, it cannot be that $p_B = q_B > 0$.

Step 2. $p_B = q_B = 0$.

Proof. Assume the contrary—i.e., without loss of generality, $q_B > p_B$. Suppose that $q_A \geq p_A$—and, by implication, that $p_R > q_R$. Thus, the expected payoff of every R-man is below 1. An R-man can thus profitably deviate to A, since by $M3^*$, he interacts and receives $1 + \varepsilon$. Therefore, $p_A > q_A$. It follows that $q_R = 0$—otherwise, any R-woman can profitably deviate to A—she interacts and receives $1 + \varepsilon$, by $M3^*$. Now, it must be that the payoff of every B-woman is at least $1 + \varepsilon$ (otherwise, she can profitably deviate to A, since by $M3^*$, she interacts and receives $1 + \varepsilon$). Recall that the B-players produce a total surplus of $2p_B$ at most, and denote by b the average equilibrium payment that extractor B makes to the B-men. Extractor B's equilibrium payoff is thus $2p_B - bp_B - $

$q_B(1 + \varepsilon)$ at most. It is nonnegative in equilibrium, so $b < (2p_B - q_B)/p_B$ < 1. If $q_B > 2p_B$, $b < 1$, so there is a B-man whose equilibrium payoff is negative, which is impossible. On the other hand, $p_B < q_B \leq 2p_B$ implies $q_B - p_B \leq N/2$. Recall that $q_B > p_B$, $p_A > q_A$, and $q_R = 0$. It follows that $p_A - q_A \leq N/2$. Thus, by $M3^*$, any B-man who deviates to A receives $1 + \varepsilon$, regardless of his position on extractor A's list. Since $b < 1$, there must be a B-man whose payoff is strictly below 1, and he finds this deviation profitable. Therefore, it must be that $p_B = q_B = 0$.

Step 3.   Since $p_B = q_B = 0$, it must be that $p_A = q_A = N$, by proposition 2. Now, if extractor B's no-interaction payments are allowed to exceed ε, then an A-player can profitably deviate to B, a contradiction. Hence, if extractor A offers $M3^*$ in equilibrium, extractor B's best-response no-interaction payments do not exceed ε.

REFERENCES

Aghion, P., and Bolton, P. (1987). ''Contracts as a Barrier to Entry,'' *Amer. Econ. Rev.* **77**, 388–401.

Biglaiser, G. (1993). ''Middlemen as Experts,'' *RAND J. Econ.* **24**, 212–223.

Biglaiser, G., and Friedman, J. (1994). ''Middlemen as Guarantors of Quality,'' *Inter. J. Ind. Organ.* **12**, 509–531.

Bloch, F., and Ryder, H. (1994). ''Two-Sided Search, Marriages and Matchmakers,'' mimeo.

Gehrig, T. (1993). ''Intermediation in Search Markets,'' *J. Econ. Manage. Strategy* **2**, 97–120.

Rubinstein, A., and Wolinsky, A. (1987). ''Middlemen,'' *Quart. J. Econ.* **102**, 581–593.

Yavas, A. (1992). ''Marketmakers versus Matchmakers,'' *J. Finan. Intermediation* **2**, 33–58.